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Numerical solution of dual-phase-lagging heat conduction model for analyzing overshooting phenomenon

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A B S T R A C T

The overshooting phenomenon based on one dimensional dual-phase-lagging heat conduction model, has been studied by numerical technique. The effect of Fourier number and $B\left(=\frac{\tau_T}{2\tau_q}\right)$ on overshooting phenomenon have been observed. The stability of the numerical scheme has been discussed and observed that the solution is unconditionally stable. The whole analysis is presented in dimensionless form.

1. Introduction

The Fourier law of heat conduction assumes that heat flux vector $\mathbf{q}(\mathbf{r},t)$ and temperature gradient $\nabla T(\mathbf{r},t)$ appears at the same time instant t and consequently implies that thermal signal propagate with an infinite speed. The infinite speed of heat propagation, implying that a thermal disturbance applied at a certain location in a medium, can be sensed immediately anywhere else in the medium [1], it is one of the drawback of Fourier's law.

Much effort has been devoted to the improvement of classical Fourier law. Cattaneo[2] and Vernotee [3] independently proposed a modified version of heat conduction equation by adding a relaxation term to represent the lagging behavior of energy transport within the solid, which takes the form.

$$\tau_{CV}\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} = -k\nabla T, \tag{1.1}$$

where, k is the thermal conductivity of medium and τ_{CV} is a material property called the relaxation time. This model characterizes the combined diffusion and wave like behavior of heat conduction and predicts a finite speed

$$V_{CV} = \left(\frac{k}{\rho c_b \tau_{CV}}\right)^{1/2}, \tag{1.2}$$

for heat propagation [4], where, ρ is the density and c_b is the specific heat capacity. This model addresses short time scale effects over a spatial macroscale. Detailed reviews of thermal relaxation in wave theory of heat propagation were performed by Joseph and Preziosi [5] and Ozisik and Tzou [6]. The natural extension of CV model is

$$\mathbf{q}(\mathbf{r}, t+\tau_{CV}) = -k\nabla T(\mathbf{r},t), \tag{1.3}$$

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Nomenclature

A	a constant
B	a constant, $\frac{\tau_T}{2\tau_q}$
c	thermal wave propagation speed, m/s
c_b	specific heat capacity, J/kg K
C	a constant
C_e	volumetric heat capacity of electron, J/m^3 K
C_l	volumetric heat capacity of lattice, J/m^{3}K
D	a constant
f_r	reference heat flux
F_0	Fourier number, $\frac{c^2 t}{2\alpha}$
ΔF_0	temporal grid size, m
G	function of electron mass, m
k	thermal conductivity, W/m K
l	length of heat conduction medium, m
P	a matrix of order $N \times N$
Q	a matrix of order $N \times N$
$q(t)$	heat flux, W/m^2
$q*$	dimensionless heat flux, $\frac{q}{f_r}$
$\mathbf{r}$	position vector
R	a matrix of order $N \times N$
t	time, s
T	temperature, K
∇T	temperature gradient, K/m
V_{CV}	thermal wave propagation speed, m/s
x	dimensionless spatial coordinate, $\frac{cy}{2\alpha}$
Δx	spatial grid size m
y	spatial coordinate, m
α	thermal diffusivity, m^2/s
θ	introducing variable, $A\bar{\theta} + \frac{\partial \bar{\theta}}{\partial F_0}$
$\bar{\theta}$	dimensionless temperature, $kcT/\alpha f_r$
ρ	density, kg/m^3
τ_q	phase lag of heat flux vector, s
τ_T	phase lag of temperature gradient, s
τ_{CV}	relaxation time, s

which is called the single-phase-lagging (SPL) heat conduction model [7]. According to SPL heat conduction model, there is a finite built-up time τ_{CV} for onset of heat flux at $\mathbf{r}$ after a temperature gradient is imposed there i.e. τ_{CV} represents the time lag needed to establish the heat flux (the result) when a temperature gradient (the cause) is suddenly imposed.

Many new simulation models such as phonon-electron interaction in metal films [8,9], phonon scattering in dielectric crystals [10], insulators and semi conductors [11–13], have recently been developed in order to study the mechanisms of heat conduction in microscale/nanoscale that cannot be described by Fourier's law. To describe micro-structural interactions a further modification of SPL model gives the dual-phase-lagging (DPL) model [1,14],

$$\mathbf{q}(\mathbf{r}, t + \tau_q) = -k\nabla T(\mathbf{r}, t + \tau_T), \tag{1.4}$$

where, τ_T is the phase lag of temperature gradient and τ_q is the phase lag of the heat flux vector. It allows either the temperature gradient (cause) to precede the heat flux vector (effect) or vice versa.

In non-Fourier heat conduction models such as CV and dual-phase-lagging heat model, the temperature of some inner regions in the medium may exceed to the temperature at the boundary, this is called the overshooting phenomenon [15]. Consequently this phenomena may lead to the damage of electronic or mechanical devices if it is not handled properly. Thus to handle such type of problem, it is very desirable to construct high order algorithms for efficient computations.

The overshooting phenomenon has been investigated by Jou and Criado-Sancho based on the extended irreversible thermodynamics [16]. It was found that initial value of temperature gradient can not be arbitrarily high in DPL heat conduction model. Based on the CV model, Al-Nimr et al. [17], found that initial first and second order time derivatives of temperature field control the occurrence of overshooting phenomenon. Mingtian et al. [15], investigated overshooting phenomenon in one dimensional DPL heat conduction model and found that the interference of thermal waves results in the overshooting of temperature field. In the present work, we attempt to examine the effect of $B = \frac{\tau_T}{2\tau_q}$ (which causes the interference) and Fourier number F_0 on overshooting phenomenon based on DPL heat conduction model.