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A no spill-over updating method for undamped structural systems

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ABSTRACT

A new numerical method for simultaneously updating mass and stiffness matrices based on incomplete modal measured data is presented. By using the Kronecker product, all the variables that are to be modified can be found out and then can be updated directly. The optimal approximation mass and stiffness matrices which satisfy the required eigenvalue equation are found under the Frobenius norm sense. The large number of unmeasured and unknown eigeninformation and the physical connectivity of the original model are preserved and the updated model will exactly reproduce the modal measured data. The method is computationally efficient as neither iteration nor eigenanalysis is required. The numerical results show that the method proposed is reliable and attractive.

1. Introduction

Using finite element techniques, the undamped free vibration of a structural dynamic system can be described by the second order differential equation as

$$M_a\ddot{q}(t) + K_a q(t) = f(t), \tag{1}$$

where $M_a,\ K_a \in \mathbf{R}^{n\times n}$ are analytical mass and stiffness matrices, $q(t)$ is the $n \times 1$ vector of positions, and $f(t)$ is the $n \times 1$ vector of external force. In general, M_a is real-valued symmetric and positive definite, and K_a is real-valued symmetric and positive semidefinite. Eq. (1) is usually known as the finite element analytical model. By considering the homogeneous part of Eq. (1) and assume that the displacement response of (1) is harmonic,

$$q(t) = x(\omega)e^{i\omega t},$$

then the structural eigenproblem can be written in the form

$$K_a x_j = \lambda_j M_a x_j, \quad j = 1, 2, \ldots, n, \tag{2}$$

where $\lambda_j = \omega_j^2$ is the jth eigenvalue and x_j is the jth eigenvector. It is well known that the eigenvalue and eigenvector can be interpreted physically as the square of the natural frequency of vibration and the mode shape respectively. Let

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}, \quad X = [X_1, X_2],$$

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where $\Lambda_1 = \text{diag}\{\lambda_1, \ldots, \lambda_p\}$, $\Lambda_2 = \text{diag}\{\lambda_{p+1}, \ldots, \lambda_n\}$, $X_1 = [x_1, \ldots, x_p]$ and $X_2 = [x_{p+1}, \ldots, x_n]$. Then it is easy to see that the n columns of the matrix equation

$$M_a X\Lambda = K_a X \tag{3}$$

summarise n separate eigenvalue–eigenvector relations of type (2). A most important property of the undamped vibration modes is their orthogonality with respect to mass, that is,

$$X^\top M_a X = I_n. \tag{4}$$

In the modern analysis of structural dynamics, much effort is devoted to the derivation of an accurate dynamic finite element model (FEM) of a structure. This accurate model is used in many applications of civil engineering structures like damage detection, health monitoring, structural control, structural evaluation and assessment. But there are some inaccuracies or uncertainties that may be associated with a finite element model. The discretization error, arising due to the approximation of a continuous structure by a finite number of individual elements, is inherent to the finite element technique. While other inaccuracies may be due to the assumptions and simplifications made by the analyst with regards to the choice of elements, modelling of boundary conditions, joints, etc. When dynamic tests are performed to validate the analytical model, inevitably their results, commonly natural frequencies and mode shapes, do not coincide well with the expected results from the analytical model. Finite element model updating is a procedure that updates the uncertainty parameters in the initial finite element model based on the experimental results so that a more realistic or refined model can be achieved.

We will assume that only a few eigenvalue and the corresponding eigenvectors (measured at full degree of freedom) are available. The reason is that in vibration industries, quantities related to high modal data in a finite-dimensional model generally are susceptible to measurement errors due to the hardwire limitations. In fact, in a large and complicated physical system, it is often impossible to acquire knowledge of the entire spectral information. While there is no reasonable analytical tool available to evaluate the entire spectral information, we can attain only partial information through experiments.

It is well known that in all physical systems the matrices are not just simply required to be symmetric, the parameters in the stiffness and mass matrices are correlated, and updating one parameter requires that others be updated in a specific fashion to maintain the proper connectivities in the structure. Although many minimization methods can reproduce the given set of measured data while updated matrices symmetry, the mass and stiffness matrices can be dramatically altered. Particularly troublesome is the modification of stiffness coefficients from values of zero to large magnitude nonzero values. Clearly, the introduction of load paths that do not exist in the actual hardware is undesirable. On the other hand, in conducting the updating, it is often desirable to match only the part of observed data without tampering with the other part of unmeasured or unknown eigenstructure inherent in the original model. Such an updating is said to be no spill-over. In this case assume that the matrices of incomplete spectral information $\Lambda_1 \in \mathbf{R}^{p\times p}$, $X_1 \in \mathbf{R}^{n\times p}$ are known for the first p eigenvalues and associated eigenvectors of the original system. The remainder of the spectral properties $\Lambda_2 \in \mathbf{R}^{(n-p)\times(n-p)}$, $X_2 \in \mathbf{R}^{n\times(n-p)}$ are not being changed and are unknown. No spill-over is required either because these high order modal data are proven to be acceptable in the original model and engineers do not wish to introduce new vibrations via updating or because engineers simply do not know of any information about these modal data.

In the past 30 years, the model updating problem has received much attention and many approaches to it have been presented. An extensive survey of model updating methods can be found in [1]. A good introductory overview of the model updating methods may be found in [2]. In early 1980s, Lagrange multiplier methods were introduced by Baruch [3] and Berman and Nagy [4]. These methods usually assumed that either the mass matrix or the stiffness matrix is correct. Then an objective function, with constraints imposed through Lagrange multipliers, is minimised in order to derive updated system matrices. The matrix mixing methods were developed by Caesar [5] and Link et al. [6]. This approach sought to combine experimental modal data with analytical ones to construct the inverses of the mass and stiffness matrices. The control-based eigenstructure assignment techniques were proposed by Zimmerman and Widengren [7] and Inman and Minas [8]. These methods determined the pseudocontrol which would be required to produce the measured modal properties with the initial structural model. The pseudocontrol was then translated into matrix adjustments applied to the FEM. These early methods are direct and computationally efficient. However, physical meanings of the updated system matrices are often not preserved and this raises the question on its validity. In order to preserve the original stiffness matrix pattern, Kabe [9], Caesar and Peter [10], Kammer [11], Smith and Beattie [12,13], Halevi and Bucher [14] and Sako and Kabe [15] developed some algorithms to preserve the connectivity of the structural model. However, these methods can not guarantee that the updated model is of no spill-over. Recently, assuming that the mass matrix is not updated, Carvalho et al. [16] proposed a direct method for undamped model updating with no spill-over. Chu et al. [17–19] considered damped model updating with no spill-over. Mao and Dai [20] developed an updating method with positive definiteness and no spill-over. A major drawback of these methods is that the updated matrices have little physical meaning and cannot be related to physical changes to the finite elements in the original model. The connectivity of nodes is not ensured and the updated matrices are fully populated, whereas the initial matrices are sparse and only contain non-zero elements in a band along the leading diagonal.

The purpose of the work presented in this paper is to develop a new direct method with no spill-over for finite element model updating problems which preserves the connectivity of the original model. Assume that M_a and K_a are real-valued symmetric $(2r+1)$-diagonal matrices. Thus, the problem of updating mass and stiffness matrices simultaneously can be mathematically formulated as following inverse eigenvalue problem and an associated optimal approximation problem.