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Analysis of composite plates using moving least squares differential quadrature method

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ABSTRACT

In this work, the moving least squares differential quadrature method (MLSDQM) is employed to analyze bending problems of composite plates. Based on a transverse shear theory, the governing equations of the problem are derived. The transverse deflection and two rotations of the plate are independently approximated with MLS approximations. The weighting coefficients used in the MLSDQ approximation are obtained through the fast computation of the MLS shape functions and their partial derivatives. The obtained results are compared with the previous analytical and numerical ones. Further a parametric study is introduced to investigate the effects of elastic and geometric characteristics on the values of transverse deflection of the plate.

1. Introduction

Owing to their superior material properties, composite plates have been extensively used in mechanical, civil, nuclear and aerospace structures. So, analysis of such plates is one of the most important problems in structural analysis. Due to the mathematical complexity of such problems, only limited cases can be solved analytically [1–3]. Finite difference, finite element, boundary element, and discrete singular convolution methods have been widely applied to solve such plate problems [4–10]. The main disadvantage of such techniques is to require a large number of grid points as well as a large computer capacity to attain a considerable accuracy [11–16].

In seeking a more efficient technique that requires fewer grid points and achieves acceptable accuracy, the differential quadrature (DQ) method is recently introduced. As stated by Civan and Sliepcevich [17], the DQM approximates the partial derivative of a variable at a given discrete point as a weighted linear sum of the function values at all the discrete points in the domain of interest. For the discontinuity problems, classical version of DQM leads to in-accurate results. This drawback can be overcomed using domain decomposition technique with DQM [18–20]. The second drawback of the classical DQM is the requirement of regular (rectangular) computational domain. Geometric mapping may be employed to transform the irregular domain of the problem to a regular one. Geometric mapping may also complicate the governing equations of the problem [21,22]. For several applications, harmonic version of DQM leads to more accurate results than that obtained by the classical version [23,24]. Ritz method or finite element method can also be combined with DQM to solve irregular plate problems [25–27].

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Liew et al. [28–32] introduced a new version of DQ based on a moving least squares approximations which is termed by moving least squares differential quadrature method (MLSDQM). The main advantage of MLSDQM is its capability to deal with discontinuity plate problems as well as irregular plate ones. The proposed technique is employed for solving several engineering problems, such as vibration and buckling problems [33–37].

The present work extends the applications of MLSDQM to analyse material discontinuity, (composite plate), problems. The shape functions and their derivatives are derived through MLS approximations for regular and irregular node patterns. The obtained results are compared with the previous analytical and numerical ones. Further a parametric study is introduced to investigate the effects of radius of support domain, order of the basis functions, sensitivity of the discrete irregular pattern, elastic and geometric characteristics of the plate on the values of the obtained results.

2. Formulation of the problem

Consider a non-homogeneous composite consisting of an isotropic plate bonded, (along x-axis), to another one made of a functionally graded material (FGM). The elastic characteristics of the composite vary such that:

$$G^f = Ge^{\gamma y}, \quad E^f = Ee^{\gamma y}, \quad v^f = v, \tag{1}$$

where G, E and v are shear modulus, Young's modulus and Poisson's ratio of the isotropic plate. G^f, E^f and v^f are shear modulus, Young's modulus and Poisson's ratio of the FG plate. γ is a constant characterizing the composite gradation.

Assume that the composite is subjected to a pure bending due to a laterally distributed load q. Based on a first-order shear deformation theory, the equilibrium equations for such plate can be written, (in tensorial notations), as [38]:

$$M_{ij,j} = Q_i, \quad Q_{i,i} = -q, \quad (i,j = x,y) \tag{2}$$

where M_{ij}, $(i,j = x,y)$, are the bending and twisting moment resultants.

Q_i, $(i = x,y)$, are the shearing force resultants.

The transverse deflection $w(x,y)$ and the normal rotations $\varphi_x(x,y)$, $\varphi_y(x,y)$ are related to the moment and shear resultants through the following constitutive relations [39].

$$M_{ij} = -D\left[\delta_{ij}(\delta_{ik}\phi_{k,k} + v(1-\delta_{kj})\phi_{k,k}) + \frac{(1-v)(1-\delta_{ij})}{2}(\phi_{i,j} + \phi_{j,i})\right], \quad (i,j,k = x,y) \tag{3}$$

$$Q_i = kGh(w_{,i} - \varphi_{,i}), \quad (i = x,y), \tag{4}$$

where $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$ and the flexural rigidity of the plate $D = \frac{Eh^3}{12(1-v^2)}$.

k is the shear correction factor [39,40], which is to be taken 5/6. h is thickness of the plate.

On suitable substitution from Eqs. (3) and (4) into (2), the equilibrium equations can be written as:

$$D\left(\frac{\partial^2\phi_x}{\partial x^2} + \frac{(1-v)}{2}\frac{\partial^2\phi_x}{\partial y^2} + \frac{(1+v)}{2}\frac{\partial^2\varphi_y}{\partial x\partial y} + \frac{(1-v)}{2}\gamma\left[\frac{\partial\varphi_x}{\partial y} + \frac{\partial\varphi_y}{\partial x}\right]\right) + kGh\left(\frac{\partial w}{\partial x} - \varphi_x\right) = 0, \tag{5-a}$$

$$D\left(\frac{\partial^2\varphi_y}{\partial y^2} + \frac{(1-v)}{2}\frac{\partial^2\varphi_y}{\partial x^2} + \frac{(1+v)}{2}\frac{\partial^2\varphi_x}{\partial x\partial y} + \gamma\left[\frac{\partial\varphi_y}{\partial y} + v\frac{\partial\varphi_x}{\partial x}\right]\right) + kGh\left(\frac{\partial w}{\partial y} - \varphi y\right) = 0, \tag{5-b}$$

$$kGh\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \gamma\frac{\partial w}{\partial y} - \frac{\partial\varphi_x}{\partial x} - \frac{\partial\varphi_y}{\partial y} - \gamma\varphi_y\right) + qe^{-\gamma y} = 0, \tag{5-c}$$

where $\gamma \neq 0$ for FG plate while $\gamma = 0$ for isotropic one.

The boundary conditions can be described (according to the case of supporting) as follows:

(a) Simply supported:

- SS1 $\quad w = 0, \quad M_n = 0, \quad M_{ns} = 0,$ (6)
- SS2 $\quad w = 0, \quad \varphi_s = 0, \quad M_n = 0,$ (7)

(b) Clamped : $\quad w = 0, \quad \varphi_s = 0, \quad \varphi_n = 0,$ (8)

(c) Free : $\quad Q_n = 0, \quad M_n = 0, \quad M_{ns} = 0,$ (9)

where the subscripts n and s represent the normal and tangent directions to the boundary edge, respectively; M_n, M_{ns} and Q_n denote the normal bending moment, twisting moment and shear force on the plate edge; φ_n and φ_s are the normal and tangent rotations about the plate edge.