Applied Mathematics and Computation 238 (2014) 329–341

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical solution for the variable order linear cable equation with Bernstein polynomials

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A R T I C L E   I N F O

Keywords:
Bernstein polynomials
Variable order fractional linear cable equation
Convergence analysis
Operational matrix
Numerical solution
The absolute error

A B S T R A C T

In this paper, Bernstein polynomials method is proposed for the numerical solution of a class of variable order fractional linear cable equation. In this paper, we adopted Bernstein polynomials basis defined on the interval $[0, R]$ to solve the equations defined on the section $\Omega = [0, X] \times [0, T]$. The main characteristic behind this approach in this paper is that we derive two kinds of operational matrixes of Bernstein polynomials. With the operational matrixes, the initial equation is transformed into the products of several dependent matrixes which can also be viewed as the system of linear equations after dispersing the variable. By solving the linear system of algebraic equations, the numerical solutions are acquired. Only a small number of Bernstein polynomials are needed to obtain a satisfactory result. Numerical examples are provided to show that the method is computationally efficient.

1. Introduction

As one of the most fundamental equations, the cable equation successfully modeled so many complex problems appearing in neuronal dynamics. From the Rall's pioneering work [1], in recent years, more and more researchers have focused on the study of neuronal dynamics [2–15]. For example, recently, Langlands et al. [8] have proposed and investigated the following fractional cable equation which can be viewed as macroscopic models for electrodiffusion of ions in nerve cells when molecular diffusion is anomalous subdiffusion due to binding, crowding or trapping

$$\frac{\partial u(x,t)}{\partial t} = D_t^{1-r_1} \frac{\partial^2 u(x,t)}{\partial x^2} - \mu D_t^{1-r_2} u(x,t) + f(x,t), \tag{1}$$

where $0 < r_1, r_2 < 1$, $\mu > 0$ is a constant, here $D_t^{1-r} g(x,t)$ is the variable-order Caputo fractional partial derivative of order $1 - r$.

In this paper, we consider the following variable order linear cable equation

$$\frac{\partial u(x,t)}{\partial t} = D_t^{1-r_1(x,t)} \frac{\partial^2 u(x,t)}{\partial x^2} - \mu D_t^{1-r_2(x,t)} u(x,t) + f(x,t), \tag{2}$$

with the initial and boundary conditions

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http://dx.doi.org/10.1016/j.amc.2014.03.066
0096-3003/

$$\begin{aligned} &u(x,0) = g(x), \quad 0 \le x \le X, \\ &u(0,t) = \phi(t), \quad u(1,t) = \varphi(t), \quad 0 \le t \le T, \end{aligned} \tag{3}$$

where $0 < r_{\min}^{(1)} \le r_1(x,t) \le r_{\max}^{(1)} < 1$, and $0 < r_{\min}^{(2)} \le r_2(x,t) \le r_{\max}^{(2)} < 1$, $\mu > 0$ is a constant here $D_t^{1-r(x,t)} g(x,t)$ is the variable-order Caputo fractional partial derivative of order $1 - r(x,t)$.

As is known to all, many numerical methods using different kinds of fractional derivative operators for solving different types of fractional differential equations have been proposed. The most commonly used ones are Adomian decomposition method (ADM) [16,17], Variational iteration method (VIM) [18], generalized differential transform method (GDTM) [19,20], generalized block pulse operational matrix method [21] and wavelet method [22,23]. Also there are some novel methods, such as the tau approach based on the shifted Legendre-tau idea [24,25] and the homotopy analysis method [26].

Since the kernel of the variable order operators is too complex for having a variable-exponent, the numerical solutions of variable order fractional differential equations are quite difficult to obtain, and have not attracted much attention. Fig. 1 Therefore, the development of numerical techniques to solve variable order fractional differential equations has not taken off. Coimbra [27] employed a consistent approximation with first-order accurate for the solution of variable order differential equations. Soon [28] proposed a second-order Runge–Kutta method which is consisting of an explicit Euler predictor step followed by an implicit Euler corrector step to numerically integrate the variable order differential equation. Sun et al. [29] introduced a classification of the variable-order fractional diffusion models to the diffusion curve of the variable order differential operator model based on the possible physical origins, which motivated the variable-order and developed the Crank–Nicholson scheme. Lin et al. [30] studied the stability and the convergence of an explicit finite-difference approximation for the variable-order fractional diffusion equation with a nonlinear source term. Fig. 2 Zhuang et al. [31] obtained explicit and implicit Euler approximations for the variable-order fractional advection–diffusion nonlinear equation. Chen et al. [32] studied a variable-order anomalous subdiffusion equation and employed two numerical schemes, one with fourth order spatial accuracy and first order temporal accuracy, the other with fourth order spatial accuracy and second order temporal accuracy. However, as far as we know, no one had attempted to seek the numerical solution of the variable order fractional equations.

So in this paper, we introduce the Bernstein polynomials to seek the numerical solution of the variable order fractional linear cable equation. With the simple structure and perfect properties, the Bernstein polynomials play an important role in various areas of engineering and mathematics. Those polynomials have been widely used in solving fractional integral equations and fractional differential equations [33–42]. In recent years, various polynomials such as Taylor series [43,44] and Legendre polynomials [45–47] have been applied to seek the numerical solution of integral, differential equations, fractional integral equations and fractional differential equations.

The reminder of the paper is organized as follows: Sections 2 and 3 are preparative, the definitions and properties of the variable order fractional order integrals and derivatives and Bernstein polynomials are given in Sections 2 and 3. In Section 4, Fig. 3 the two kinds of operational matrixes of Bernstein polynomials are derived and we applied the operational matrixes to solve the equation as given at beginning. In Section 5, we present some numerical examples to illustrative the method and to demonstrate efficiency of the method. We end the paper with a few concluding remarks in Section 6.

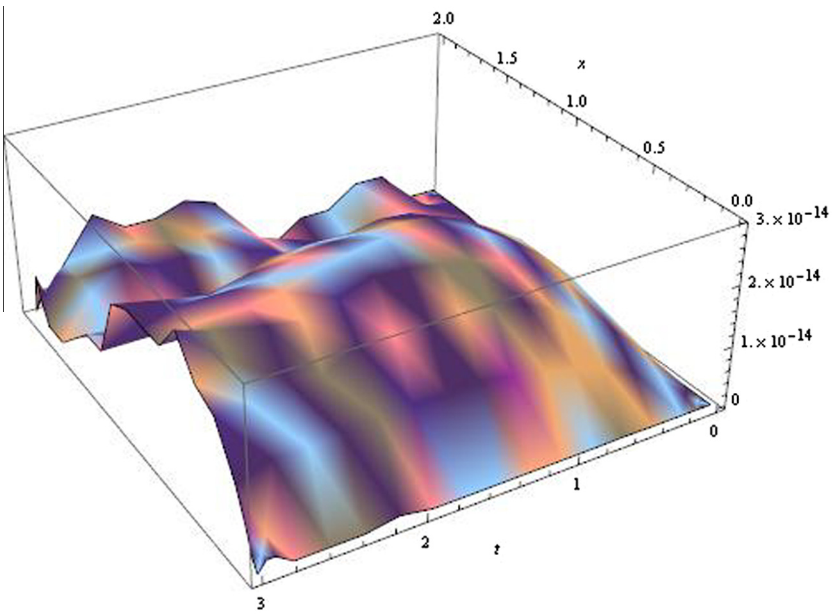

Fig. 1. The absolute error for Example 1 when $n = 3$.