Applied Mathematics and Computation 238 (2014) 358–369

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

New stability and stabilization results for discrete-time switched systems

Ruihua Wang *, Shumin Fei

School of Automation, Southeast University, Nanjing 210096, Jiangsu, People's Republic of China

ARTICLE INFO

Keywords:
Switched systems
Exponential stability
Forward average dwell time
Forward mode-dependent average dwell time
Mode-dependent controllers

ABSTRACT

In this paper, the stability and stabilization problems are considered for discrete-time switched systems in both nonlinear and linear contexts. By introducing the concept of forward average dwell time and using multiple-sample Lyapunov-like functions variation, the extended stability results for discrete-time switched systems in the nonlinear setting are first derived. Then, the criteria for stability and stabilization of linear switched systems are obtained via a newly constructed switching strategy, which allows us to obtain exponential stability results. A new kind of mode-dependent controller is designed to realize the stability conditions expressed by the multiple-sample Lyapunov-like functions variation. Based on this controller, the stabilization conditions can be given in terms of linear matrix inequalities (LMIs), which are easy to be checked by using recently developed algorithms in solving LMIs. Finally, a numerical example is provided to show the validity and potential of the results.

1. Introduction

Switched systems are composed of continuous-time or discrete-time systems with (isolated) discrete switching events, which are triggered by switching mechanisms. Switched systems have strong engineering background in various areas and are often used as a unified modeling tool for a great number of real-world systems such as power electronics, chemical processes, mechanical systems, automotive industry, aircraft and air traffic control and many other fields. Therefore, lots of efficient methodologies have been proposed in the literature to deal with the stability and stabilization problems for switched systems [1,2,4–7,9,10,13,16]. One way is "dwell time", which, then, is extended to the concept of "average dwell time (ADT)" for more flexibility and availability in system analysis and control synthesis; see [3,4,10,12,13,15,16] and the references therein. By allowing the Lyapunov-like function of every subsystem to increase with a finite increase rate and in less than a constant time during its running time, the extended stability results were obtained for continuous-time and discrete-time switched systems with ADT in nonlinear setting [12]. Recently, a new concept, "mode-dependent ADT", has been introduced by Zhao et al. [16], where each mode in the underlying switched system has its own ADT. The increase coefficient of the Lyapunov-like function at switching instants and the decay rate of the Lyapunov-like function during the running time of subsystems are set in a mode-dependent manner, which reduces the conservativeness in the results.

* Corresponding author.
E-mail address: 123.wrh@163.com (R. Wang).

http://dx.doi.org/10.1016/j.amc.2014.04.016
0096-3003/

Multiple-sample Lyapunov functions variation has been used to investigate stability and stabilization for fuzzy systems and switched systems [17,18,20,21]. In [17], the stabilization problem for a class of uncertain discrete-time Takagi–Sugeno fuzzy systems was studied through a nonquadratic Lyapunov function. k-sample Lyapunov functions variation, i.e., $\Delta_k V(x(t)) = V(x(t+k)) - V(x(t))$, is put in use to design a robust control law such that the close-loop system is globally asymptotically stable. This paper [18] proposed a new approach for the stability analysis and controller synthesis of discrete-time Takagi–Sugeno fuzzy dynamic systems. In this paper non-monotonic Lyapunov function is utilized to relax the monotonic requirement of Lyapunov theorem which renders a larger class of functions to provide stability, which decreases every few steps; however, can be increased locally. The authors [20] gave two new sufficient conditions of global asymptotic stability for nonlinear systems that allow the Lyapunov functions to increase locally, but guarantee an average decrease every few steps. In order to demonstrate the strength of their methodology, it was shown that tighter bounds on the joint spectral radius can be obtained using common non-monotonic Lyapunov functions for discrete-time linear switched systems. In [21], the stability problem was studied for discrete time linear switched systems based on minimum dwell time by means of a family of quadratic Lyapunov functions. The inequality $(A_i^{\triangle})^T P_j A_i^{\triangle} - P_i$, where $\triangle$ is the lower bound for dwell time, was introduced to obtain asymptotic stability for linear switched systems. However, the inequality $A_i^T P_i A_i - P_i < 0$ is also necessary therein, which denotes that every subsystem is required to be asymptotically stable. Besides, because $(A_i^{\triangle})^T P_j A_i^{\triangle} - P_i$ is nonconvex in A_i, the authors didn't find efficient controller design methods. Inspired by these results, we will employ multiple-sample Lyapunov-like functions variation to establish new mode-dependent controllers (multiple controllers for each mode in one switching interval), guaranteeing that the closed-loop system is exponentially stable.

The remaining of the paper is organized as follows. In Section 2, necessary preliminary knowledge is given and the considered problem is formulated. Section 3 is devoted to the main results of the paper. By using multiple-sample Lyapunov-like functions variation and a new concept–forward ADT, the sufficient conditions ensuring the asymptotic stability of nonlinear discrete-time switched systems are firstly derived. Then, new mode-dependent controllers are constructed to guarantee the exponential stability of the resulting closed-loop system for linear switched systems. In Section 4, a powerful example is provided to show the potential and the validity of the obtained results. In the end, concluding remarks are given in Section 5.

Notations. The notations in this paper are fairly standard. We use $A > 0$ ($A < 0$) to stand for a positive definite (negative definite) matrix A. A^T denotes the transpose of a matrix A, and $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) represents the maximum (respectively, minimum) eigenvalue of A. The set of symmetric matrices of size n is denoted by S^n. Let $\mathbb{Z}_{\geqslant 0}$ denote the set of nonnegative integers, and $\mathbb{R}^n$ mean the n-dimensional Euclidean space. As is commonly used in other literature, $*$ denotes the elements below the main diagonal of a symmetric matrix, and max and min, respectively, stands for the maximum and minimum. $\mathcal{C}^1$ denotes the space of continuously differentiable functions, and a function $\rho : [0,\infty) \to [0,\infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\rho(0) = 0$. In addition, $\|\cdot\|$ is used to denote the vector Euclidean norm, and I refers to an identity matrix of the appropriate dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation and preliminaries

Consider a class of discrete-time switched linear systems described by

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \quad k \geqslant k_0, \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state, and $u(k) \in \mathbb{R}^{n_u}$ is the control input. $\sigma(k)$ is a piecewise constant function of time, called a switching signal, which takes its values in a finite set $\underline{N} = \{1, 2, \ldots, N\}$; $N > 1$ is the number of subsystems. A_i and B_i are constant real matrices with appropriate dimensions for any $i \in \underline{N}$.

For a piecewise constant switching signal $\sigma(\tau)$, let $k_1 < k_2 < \cdots < k_m < \cdots$ denote the switching instants of $\sigma(\tau)$ and $\{x(k_0); (i_0, k_0), (i_1, k_1), \ldots, (i_m, k_m), \ldots\}$ denote the switching sequence, which means that the i_mth subsystem is activated when $k_m \leqslant \tau < k_{m+1}$, or equivalently, $\sigma(\tau) = i_m$ when $k_m \leqslant \tau < k_{m+1}$.

For the purpose of the paper, the definitions of the ADT property used to restrict a class of switching signals are recalled as follows.

Definition 1 [13]. For any $k \geqslant k_0 \geqslant 0$ and any switching signal σ, let N_σ denote the number of switchings of σ over the interval $[k_0, k)$. For given constants $T_a > 0$, $N_0 \geqslant 0$, if the inequality $N_\sigma \leqslant N_0 + (k - k_0)/T_a$ holds, then the positive constant T_a is called the ADT and N_0 is called the chattering bound.

Definition 2 [16]. For a switching signal σ and any $k \geqslant k_0 \geqslant 0$, let $N_{\sigma i}(k, k_0)$ be the number that the ith subsystem is activated over the interval $[k_0, k)$ and $T_i(k, k_0)$ denote the total running time of the ith subsystem over the interval $[k_0, k)$, $\forall i \in \underline{N}$. For given constants $T_{ai} > 0$, $N_{0i} \geqslant 1$, if the inequalities $N_{\sigma i}(k, k_0) \leqslant N_{0i} + T_i(k, k_0)/T_{ai}$ hold, then the positive constant T_{ai} is called the mode-dependent ADT and N_{0i} is called the mode-dependent chattering bound.