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# Mathematical study of stage-structured pests control through impulsively released natural enemies with discrete and distributed delays

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#### ABSTRACT

Many insect pest species have two major stages in their life-history: an immature larval stage and a mature adult stage. The immature larval stage may have a significant duration, during which the insect may not be the same natural predators as in the adult stage. Keeping this biological background in mind, we propose a stage-structured natural enemy–pest model with distributed and discrete delays for natural enemy and pest, respectively. It is also assumed that the natural enemies are released in an impulsive manner. The global attractivity of the pest-extinction periodic solution of the system is obtained using comparison techniques on impulsive delay differential equations. Furthermore, the permanence conditions for the coexistence of the species are explored, which indicates that the maturation time delay and the amount of natural enemies released in a pulse can affect the dynamics of the system. The threshold values for maturation delay of pest and the releasing amount natural enemy are obtained for extinction and permanence of the species. Finally, numerical simulation is given in support of validation of the theoretical results to provide a strategic basis for the biological pest management.

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# 1. Introduction

In agriculture, effective pest control is a major problem for the farmers. Pests can be controlled by many methods such as biological, physical, or chemical control. It is a well known fact that pest is a harmful insect and its outbreak often causes serious ecological and economic problems [1,2]. Evidence indicates that annually the pests cause 25% loss in rice, 5–10% in wheat, 30% in pulses, 35% in oilseeds, 20% in sugarcane and 50% in cotton [3]. With the development of society and the progress in science and technology, man has adopted some advanced and modern weapons, such as chemical and biological pesticides [4,5], for pests control. In some cases satisfactory achievement that have been reported in the literature on pest control. A large variety of chemical pesticides were used to control pests, because they can quickly kill a significant portion of a particular pest population and sometimes provide the only feasible method for preventing economic loss. However, excessive uses of pesticides increase environmental pollution and is recognized as a major health hazard to human beings as

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http://dx.doi.org/10.1016/j.amc.2014.04.029 0096-3003/© 2014 Elsevier Inc. All rights reserved. well as other beneficial insects. Millions of people are poisoned by pesticides every year, and pest problems are often made worse when the balance between beneficial and harmful insects is disturbed by applying toxic chemicals [1,2].

Thus, biological control can be an alternative of chemical control method. Biological control suppresses the pest populations with the help of other living organisms, often called natural enemies or beneficial species [4–6]. This method may be considered successful if either the pest population is extinct or maintains at adequately low levels. Specifically, the intention here would be to suppress the abundance of the pest in a new target region to a level at which it no longer causes economic damage. This can be achieved by augmentation, which is exploitation of existing natural enemies to increase their effectiveness, and can be accomplished by mass production and periodic release of natural enemies for the pest [4].

Furthermore, many evolutionary processes are characterized by the fact that at certain moments of time they experience a change of state abruptly; these processes are subject to short-term perturbations [7–9]. Moreover, time delay factor has a great importance in population dynamics. Mainly, two types of time delay have been used by the earlier researchers, namely, discrete and distributed delays. In the last few decades, different types of models with discrete and distributed delays have been investigated by many researchers [10–20]. It has been found that continuously distributed delay models are more realistic [16] and more accurate than instantaneous time lags [17]. In this paper, we introduce a distributed delay in a natural enemy–pest model.

Since most of the insect populations have two major stages in their life cycle: immature (larval) and mature (adult). In [21], authors have studied a stage-structured single species model, assuming an average age to maturity (i.e., as a discrete time delay) which reflects a delayed birth of immature and a reduced survival of immature to their maturity with following system of delay differential equations:

$$\dot{\mathbf{x}}(t) = \beta \mathbf{y}(t) - r\mathbf{x}(t) - \beta e^{-r\tau} \mathbf{y}(t-\tau),$$
  

$$\dot{\mathbf{y}}(t) = \beta e^{-r\tau} \mathbf{y}(t-\tau) - \eta \mathbf{y}^2(t),$$
(1.1)

where x(t) and y(t) are respectively the immature and mature populations densities,  $\tau$  represent a constant time to maturity. In this model, it was assumed that at any time t > 0, the growth rate of immature population is proportional to the existing mature population with proportionality constant  $\beta$ , the death rate of immature population is r and the death rate is proportional to the square of the population with the proportionality constant  $\eta$ . Further, the term  $\beta e^{-r\tau}y(t - \tau)$  represents the immature who were born at time  $t - \tau$  and survive at time t and therefore it represents the amount of transformation from immature to mature. Here all the parameters  $\tau$ ,  $\beta$ , r and  $\eta$  are positive constants.

In recent years, the effect of impulsive perturbation on ecological system with distributed delay have been studied by many researchers [22–25,7,26]. Guo and Chen have studied the dynamics of impulsive harvest of predator using following predator–prey model with distributed delay [26]:

$$\begin{aligned} \dot{x}(t) &= x(t)(r_1 - a_1 x(t) - b_1 y(t)), \\ \dot{y}(t) &= y(t) \left( r_2 - a_2 y(t) + b_2 \int_{-\infty}^{t} F(t - s) x(s) ds \right), \end{aligned} \right\} t \neq nT, \\ \Delta x(t) &= x(t^+) - x(t) = 0, \\ \Delta y(t) &= y(t^+) - y(t) = -Ey(t), \end{aligned} \} t = nT, \ n \in N,$$

$$(1.2)$$

where x(t) and y(t) are the population densities of prey and predator;  $r_1$ ,  $r_2$  are the intrinsic growth rates;  $a_1$ ,  $a_2$  are the rates of intra-specific competition of the prey and the predator, respectively. Again,  $b_1$  is the per-capita rate of predation and  $b_2$  denotes the product of the per-capita rate of predation and the rate of conversion. The kernel function F(t) satisfies  $\int_{0}^{+\infty} F(s) ds = 1$ . Here, T is the period of impulsive harvest; 0 < E < 1 is the proportion of harvest at fixed moments nT,  $n \in N$ .

Moreover, in the real world, all insect and mite pests have some natural enemies which can effectively suppress pests and their interaction can be treated as a prey-predator interaction. For example, *stem borers* are serious pest for rice crops in India, which damage 20–50% of crops. This pest has broadly two life stages: immature (i.e., egg, larvae and pupa) and mature (adult) stage, where the immature species cannot produce new offspring. *Bats* and *spiders* are the major natural enemies of mature *stem borers*, i.e., they attack on *mature stem borers*, but they cannot attack on immature due to its invisible size [27].

The population model (1.2) in the literature ignores the reality of the stage-structure of pest and assumed that all individuals are identical. However, in many situations stage-structure can influence population size and growth in a major way. Drawing on this motivation in this paper we extend the natural enemy–pest model (1.2) using stage-structure for pest and augmentation approach of biological control. This paper is organized in six sections: the proposed mathematical model is given in Section 2. In Section 3, some preliminaries and important lemmas for positivity and boundedness are discussed. The global attractivity of pest-extinction periodic solution and permanence of the system are discussed in Section 4. Section 5 deals with numerical simulations and finally conclusions and discussions are given in the last section.

### 2. Proposed mathematical model

Motivated by Aiello and Freedman [21], Guo and Chen [26] and Birthal and Sharma [27], in this paper, we combine the modeling ideas of (1.1) and (1.2) and propose a natural enemy-pest model with discrete and distributed delays under the following assumptions:

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