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Error analysis for moving least squares approximation in 2D space

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ABSTRACT

In this paper, we provide a theoretical analysis of the moving least squares (MLS) approximation, which belongs to the family of meshless methods. First the non matrix form of the MLS shape function in two-dimensional space is obtained by using consistency conditions. The error estimates for MLS approximation in Sobolev space are presented when $u(x,y) \in C^{m+1}(\Omega)$, and $u(x,y) \in W^{m+1,q}(\Omega)$, respectively. We establish the error estimates for interpolating element-free Galerkin (IEFG) method when it is used for solving Poisson's equation. The error bound is related to the radii of the weight functions and the bound of the norm of derivatives of shape functions. Three numerical examples are selected to confirm our analysis.

1. Introduction

In recent years, meshless methods have been developed as alternative numerical approaches in effort to eliminate known shortcomings of the mesh-based methods [1]. The main advantage of these methods is to approximate the unknowns by a linear combination of shape functions. Shape functions are based on a set of nodes and a certain weight function with a local support associated with each of these nodes. Therefore, they can solve many engineering problems that are not suited to conventional computational methods [2–8].

Meshless methods contain two steps: construction of shape functions and their derivatives, and meshless discretization of governing partial differential equations. Many kinds of meshless methods have been developed, such as smoothed particle hydrodynamics (SPH) [9], diffuse element method (DEM) [10], element-free Galerkin (EFG) method [11], reproducing kernel particle method (RKPM) [12], finite point method (FPM) [13], meshless local Petrov–Galerkin (MLPG) method [14], point collocation method (PCM) [15], radial basis functions (RBF) [16], meshless finite element method (MFEM) [17], complex variable meshless method (CVMM) [18], boundary node method (BNM) [19], local boundary integral equation (LBIE) method [20], boundary radial point interpolation method (BRPIM) [21], and boundary element-free method (BEFM) [22].

Different kinds of techniques have been used for the construction of meshless shape functions. The moving least-squares (MLS) approximation is one which is most widely used [23]. MLS approximation is a standard approach to find the best continuous function matching from a set of point values by minimizing the sum of the squares of the weighted residuals of all data points, so the MLS approximation is able to obtain a very precise solution, now it is an important method to form the shape function in meshless methods.

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Based on the MLS approximation, Belytschko et al. proposed the element-free Galerkin (EFG) method [11]. Mukherjee made an improvement on the MLS approximation in order to deal with boundary conditions conveniently in the EFG method [24]. Dai discussed an improved local boundary integral equation method for two-dimensional potential problems based on the improved MLS approximation [25].

A disadvantage of the MLS approximation is that the final algebraic equations system is sometimes ill-conditioned. Thus, sometimes we cannot obtain a satisfying solution, then improved MLS approximation was presented by Liew et al. [26]. In the improved MLS approximation, the algebraic equation system is not ill-conditioned, and can be solved without the inverse matrix. Combining the boundary integral equation method with the improved MLS approximation, the boundary element-free method was presented to solve elasticity, elastodynamics, and fracture [27–31]. The improved element-free Galerkin method based on the improved MLS approximation was discussed by Zhang et al. for the biological population problems and geometrically nonlinear analysis [32–40].

The complex variable moving least squares (CVMLS) approximation, which is an approximation of a vector function, has been developed by Liew et al. [41], and the corresponding complex variable meshless methods are presented [8,42–44]. In the CVMLS approximation the trial function of a two-dimensional problem is formed with a one-dimensional basis function, the number of unknown coefficients in the trial function of the CVMLS approximation is less than that in the MLS approximation. Combining the CVMLS approximation with the EFG method, the complex variable element-free Galerkin (CVEFG) method for two-dimensional elasticity and elastoplasticity problems were presented [45,46]. Combining the CVMLS approximation with boundary integral equation method, the complex variable boundary element-free method for two-dimensional elastodynamics problems was presented [47]. Ren et al. discussed complex variable interpolating moving least squares methed [48].

Since the shape function of the MLS approximation does not have the properties of Kronecker Delta function, the meshless method based on it must use other methods to impose essential boundary conditions, which makes the Galerkin weak form more complicated, therefore, it is important to study the interpolating MLS method. Based on the MLS approximation, Lancaster proposed interpolating moving least squares (IMLS) method [23]. By using the IMLS method, Kaljevic presented the improved EFG method in which the essential boundary condition can be applied directly [49]. An improved interpolating EFG method with nonsingular weight function was discussed by Wang et al. [50]. Ren et al. proposed an improved IMLS method, and based on it the interpolating EFG method and the improved boundary element-free method are presented [51–53].

In recent literature, the researches of the mathematical theory of meshless methods are much less than those of their applications. Liu presented MLS reproducing kernel methods and its convergence [54]. Wendland obtained error estimates for MLS approximation by using norming sets [55]. Armentano presented error estimates for MLS approximation in Sobolev spaces [56]. Zuppa obtained error estimates for derivatives of shape function by the condition numbers of the star of nodes in the normal equation [57]. Error estimate of the reproducing kernel particle method was established by Han [58]. Cheng carried out an error estimation and convergence analysis of the finite point method [59,60]. Li obtained error estimates for MLS approximation when nodes and weight functions satisfy certain conditions [61]. Salehi presented the MLS radial reproducing kernel particle method and established the convergence rate of the approximation [62].

In this paper, firstly the non matrix form of the MLS shape function in two-dimensional space is obtained by using consistency conditions. The error estimates for MLS approximation in Sobolev space are presented. We establish the error estimates for interpolating element-free Galerkin (IEFG) method when it is used for solving Poisson's equation. Three numerical examples are selected to confirm our analysis and to demonstrate the efficiency and accuracy of IEFG method.

2. Moving least squares approximation

In the MLS approximation, a function $u(\boldsymbol{x})$, $(\boldsymbol{x} \in D)$ is to be approximated, it is assumed that its values $u_I = u(\boldsymbol{x}_I)$, $(I = 1, 2, \ldots, N)$ are given.

A approximating function of $u(\boldsymbol{x})$ is

$$u^h(\boldsymbol{x}) = \sum_{i=1}^{m} p_i(\boldsymbol{x}) a_i(\boldsymbol{x}) = p^T(\boldsymbol{x})\mathbf{a}(\boldsymbol{x}), \tag{1}$$

where $p_i(\boldsymbol{x})$ are monomial basis functions, the unknown parameters $a_i(\boldsymbol{x})$ vary with x $(i = 1, 2, \ldots, m)$.

In two dimensional space, the basis functions are:

Linear basis:

$$\boldsymbol{p}^T(\boldsymbol{x}) = (1, x, y). \tag{2}$$

Quadratic basis:

$$\boldsymbol{p}^T(\boldsymbol{x}) = (1, x, y, x^2, xy, y^2). \tag{3}$$

We define a functional