Available online at www.sciencedirect.com
ScienceDirect
Optical Switching and Networking 4 (2007) 157–172
Optical Switching and Networking
www.elsevier.com/locate/osn

Review

Comparison of ILP formulations for the RWA problem

B. Jaumard[a,*], C. Meyer[b], B. Thiongane[c]

[a] *CIISE, Concordia University, Montréal, H3G 1M8, Canada*
[b] *GERAD, HEC Montréal, H3T 2A7, Canada*
[c] *CRI, Institut des Sciences de l'Ingénieur, Dakar, Sénégal*

Received 4 January 2007; accepted 25 May 2007
Available online 27 June 2007

Abstract

We present a review of the integer linear programming (ILP) formulations that have been proposed for the routing and wavelength assignment problem in WDM optical networks assuming asymmetrical traffic. We show that all formulations proposed under asymmetrical traffic assumptions, both link and path formulations, are equivalent in terms of the upper bound value provided by the optimal solution of their linear programming relaxation, although their number of variables and constraints widely differ. We propose improvements for some of the formulations that result in further reductions in the number of variables and constraints.

Under the objective of minimizing the blocking rate, we propose an experimental comparison of the best lower and upper bounds that are available. We then discuss the easiness of exact ILP solution depending on the formulations. We observe that LP relaxation bounds often provide solutions with a value very close to the optimal ILP one. We solve exactly for the first time several RWA (Routing and Wavelength Assignment) realistic instances, including those proposed by Krishnaswamy and Sivarajan [R. Krishnaswamy, K. Sivarajan, Algorithms for routing and wavelength assignment based on solutions of LP-relaxation, IEEE Communications Letters 5 (10) (2001) 435–437], with a proof of the optimality.

Keywords: WDM network; Network dimensioning; RWA problem; Integer programming; Bounds; Hop constraints; Optimal solution

Contents

1. Introduction 158
2. Statement of the RWA problem — notation 159
3. Link ILP formulations 159
   3.1. First formulation of Krishnaswamy and Sivarajan 160
   3.2. Second formulation of Krishnaswamy and Sivarajan 161
   3.3. Comparison of the LP Relaxations of the Formulations (RWA) and (RWA_sd) 162
   3.4. Source formulations 164
   3.5. Comparison of the three link formulations 167
4. Path ILP formulations 167

* Corresponding author. Tel.: +1 514 848 2424; fax: +1 514 848 3171.
E-mail address: bjaumard@ciise.concordia.ca (B. Jaumard).

1573-4277/$ - see front matter
doi:10.1016/j.osn.2007.05.002

4.1. PATH formulation 167
4.2. A relaxation of the PATH formulation 168
4.3. Comparison of the upper bounds provided by the path formulations 168
5. Further comparisons and hop constraints 169
6. Computational experience 169
7. Conclusion 171
Acknowledgements 171
References 171

1. Introduction

The WDM (Wavelength Division Multiplexing) optical networks offer the promise of providing the high bandwidth required by the increasing multimedia communication applications, see, e.g., Ramaswami and Sivarajan [1] or Mukherjee [2] for a general reference on optical networks. This has led to a wide interest in the RWA (Routing and Wavelength Assignment) problem defined as follows: given the physical structure of a network and a set of requested connections, select a suitable routing path and wavelength for each connection so that no two paths sharing a link on the same fiber, are assigned the same wavelength.

Many papers have already appeared on the RWA problem, proposing various heuristic scheme solutions under different assumptions on the traffic patterns, availability of the converters and objectives, cf. the surveys of Dutta and Rouskas [3] and Zang, Jue and Mukherjee [4] for a summary of the works until 2000, and Jaumard, Meyer and Thiongane [5] for a recent survey on symmetrical systems under various objectives. The most often studied objectives are the minimization of the number of wavelengths (called min-RWA problem), the maximization of the number of granted connections (called max-RWA problem) (Krishnaswamy and Sivarajan [6]), the minimization of the congestion (Krishnaswamy and Sivarajan [7]) and the minimization of the multiplexing costs.

Several types of heuristics and metaheuristics have been proposed. For the most efficient or recent ones, see, e.g., various greedy heuristics (Banerjee and Mukherjee [8], Banerjee, Yoo and Chen [9], Chlamtac, Ganz and Karmi [10], Zhang and Acampora [11]) and different metaheuristics: Tabu Search (Dzongang, Galinier and Pierre [12], Jaumard, Meyer and Yu [13] for nonuniform traffic, Noronha and Ribeiro [14] for uniform traffic), Simulated Annealing (Katangur, Pan and Fraser [15]) or genetic algorithms (Ali, Ramamurthy and Deogun [16]; Qin, Siew and Li [17]; Banerjee, Mehta and Pandey [18]). The reader can refer to Hyytiä [19] for a comparison of some of them.

With respect to exact solutions, the RWA problem has been formulated as an integer programming problem but most of the time those formulations have not been used for developing solution schemes except for some rounding off procedures. However, as we will see later in this paper, if a proper ILP formulation is sought, it can be used to solve very efficiently realistic RWA instances. We review those mathematical formulations for static traffic models, focusing on the max-RWA problem with asymmetrical traffic matrices. There are two classes of formulations, those with link variables (see [6,20,21]), and those with path variables (see [22–24]). We compare the optimal values of their linear programming relaxations (or LP relaxations for short) and show that they all lead to the same upper bound. We also propose some further improvements for some of the formulations that eliminate potential looping lightpaths and lead to significant reductions in the number of variables and constraints. The ease of solving different integer linear programming (or ILP for short) formulations varies very much with the number of variables and constraints: while some of the ILP formulations can be solved using ILP software (e.g. with the CPLEX libraries of ILOG inc. [25]), others are just intractable as soon as the size of the instances is increasing. This is the case for path formulations if all potential paths are considered (Ramaswami and Sivarajan [24], Lee, Lee and Park [26]), unless appropriate column generation techniques are used (Jaumard, Meyer and Thiongane [27]). Other attempts also include Lagrangean relaxation (Saad and Luo [23]; Zhang, Yang and Liu [28]). Moreover, if additional constraints need to be considered such as hop or signal regeneration constraints (Ye et al. [29], Ali, Ramamurthy and Deogun [30]), some formulations are much easier to adapt than others.

We next compare the upper bounds provided by the various LP relaxations with the optimal ILP solutions obtained using the CPLEX-MIP software of ILOG [25]. Indeed, using the ILP formulation with the smallest number of variables and constraints, we solve several realistic RWA instances exactly for the first time, with a proof of optimality, including the instances of