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Existence of multiple sliding segments and bifurcation analysis of Filippov prey–predator model

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ABSTRACT

In a prey–predator model, an effective management strategy called the threshold policy control (TPC) is proposed, resulting in Filippov system. Considering the limitation of environmental resources, Filippov system with the weighted sum as the index is put forward in the prey–predator model with Ivlev's function response. Firstly, the existences of sliding segments in four cases have been discussed completely. Further, the sliding mode dynamics, the existences of different types of equilibria and tangent point, regular/virtual equilibrium and sliding mode bifurcations have been addressed. Moreover, the results obtained in present work indicate that the local sliding bifurcations such as boundary focus, boundary node and tangent bifurcations occur sequentially with the threshold value varying. Finally, some global sliding bifurcations including touching and buckling bifurcations are investigated by employing numerical techniques.

1. Introduction

Our ecological environment is a huge and highly complex system which arises not only from the diversity of biological species, but also from the complexity of every individuality. The relationship between predator and their prey has long been and will continue to be one of significant field in mathematical ecology owing to its universal existence and importance [1,2]. Predator's functional response which refers to the change in the density of prey attached per unit time per predator as the prey density changes is an important component of the prey–predator relation. There are several famous functional response types: Holling type I-III [3,4], Hassell–Varley type [5], Beddington–DeAngelis type [6,7], Crowley–Martin type [8], Ratio-dependence type [2,9] and so on. They are all monotonically increasing and uniformly bounded functions in the first quadratic. Another functional response was suggested by Ivlev [10]: $p(x) = b(1 - e^{-ax})$, where x is the density of prey, constants a and b represent the decrease in motivation to hunt and the maximum rate of predation, respectively. Based on the Ivlev's function response, ordinary differential equations (ODE) models have been extensively studied recently by Kooij and Zegling [11], Sugie [12] and Xiao [13]. For example, the existence of limit cycle and global dynamics of the system have been addressed.

Biological strategy (releasing beneficial natural enemies), culture strategy (catching or harvesting pest artificially) and chemical strategy (spaying insecticide) are three important tactics to pest control. Integrated pest management (IPM), a long term management strategy [14–16], combines above three tactics so as to reduce pest populations to tolerable levels, i.e. the economic injury level (EIL). Threshold policy control (TPC) can be comprehended that when the number of pests reaches or

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exceeds the economic threshold (ET), control strategies should be carried out, until it falls below the ET. They have attracted great attention in agriculture, forestry, animal husbandry and so on. Based on IPM strategies, either fixed moment or state-dependent impulsive prey–predator models with Ivlev's response function have been studied by Liu et al. [17], Xiang and Song [18] and Wang and Wang [19]. However, there may be some disadvantages of the impulsive differential equation models [20]. For example, in the fixed moment impulsive models, control strategies are invariably implemented, without considering whether the density of prey reaches the ET or not, which leads to consume vast resources. Besides, in the state-dependent impulsive differential models, control strategies are carried out instantaneously, without considering that all kinds of control strategies must last certain time period and cannot be finished instantaneously.

In practice, given the limitation of environmental resources, a threshold policy called weighted escapement policy (WEP) is proposed [21–23]. The WEP prevents the density of prey (denoted by x) and predator (denoted by y) from exceeding the EIL in deterministic environment. Weighted sum $W(x,y) \triangleq \alpha_1 x + \alpha_2 y$ has been chosen as an index for farmers to implement control strategies in appropriate time, where α_1 and α_2 are attributed population weights. Control strategies should be applied only when the weighted sum $W(x,y)$ of prey and predator reaches the ET.

Consequently, it is necessary to improve the above models to describe the reality such that non-instantaneous control and a threshold policy named WEP are implemented in the model when weighted sum $W(x,y)$ of prey and predator reaches the ET. Filippov systems being a class of piecewise smooth system of differential equations with a discontinuous right-side have many applications in science and engineering, including harvesting thresholds, oilwell drilling and liquid–gas reactions [24–28], since they make the differential equation extended to a differential inclusion and provide a natural and rational framework for many real world problems. So in this paper, we extend the existing models on prey–predator model with Ivlev's functional response to be the Filippov system by considering non-instantaneous control interventions and WEP.

The organization of this paper is follows. In Section 2, we first propose the Filippov system, and give some basic definitions and preliminaries. In Section 3, choosing parameter P_2 as bifurcation parameter, we investigate the existences of sliding segments in four cases for Filippov system (2.1). The results show that the system has at most three pieces of sliding segments. In Section 4, the sliding mode dynamics, four types of equilibria and tangent point of system (2.1) are investigated. In Section 5, we consider regular/virtual equilibrium and sliding mode bifurcations of system (2.1). In Section 6, abundantly local sliding bifurcations including boundary focus, boundary node and tangent bifurcations occur sequentially with ET varying. Moreover, some global sliding bifurcations such as touching bifurcation and buckling bifurcation are studied by numerical methods with the value of ET varying. In last section, we give some discussions.

2. Filippov system and preliminaries

2.1. Filippov prey–predator model

Based on the non-instantaneous control and the limitation of energy environmental resources, Filippov prey–predator model with the weighted sum as its index is employed to describe the relationship between prey and their predators as follows

$$\begin{cases} \dot{x}(t) = rx\left(1-\frac{x}{k}\right) - b(1-e^{-ax})y - \epsilon p_1 x, \\ \dot{y}(t) = \beta b(1-e^{-ax})y - \delta y - \epsilon p_2 y + \epsilon q y, \end{cases} \tag{2.1}$$

with

$$\epsilon = \begin{cases} 0, & W(x,y) < ET, \\ 1, & W(x,y) > ET, \end{cases}$$

where x and y represent the density of prey and predator, respectively, and weighted sum $W(x,y) \triangleq \alpha_1 x + \alpha_2 y$. Constant k is the carrying capacity of prey, r and δ are intrinsic growth rate of prey and intrinsic mortality rate of predator, and β denotes the conversion rate of prey captured by predator. We assume that prey and predator are catched or transferred (culture strategy) or poisoned (chemical strategy) as proportion p_1 and p_2, respectively. The proportion of predator is released (biological strategy) as q.

When the weighted sum $W(x,y) < ET$, it is not necessary to implement the IMP strategies, then system (2.1) is determined by the following subsystem

$$\begin{cases} \dot{x}(t) = rx\left(1-\frac{x}{k}\right) - b(1-e^{-ax})y \triangleq P_1(x,y), \\ \dot{y}(t) = \beta b(1-e^{-ax})y - \delta y \triangleq Q_1(x,y). \end{cases} \tag{2.2}$$

If and only if the weighted sum $W(x,y) > ET$, the IMP strategies must be applied in time, then we have the following control mode

$$\begin{cases} \dot{x}(t) = rx\left(1-\frac{x}{k}\right) - b(1-e^{-ax})y - p_1 x \triangleq P_2(x,y), \\ \dot{y}(t) = \beta b(1-e^{-ax})y - \delta y - p_2 y + q y \triangleq Q_2(x,y). \end{cases} \tag{2.3}$$