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New and improved results on stability of static neural networks with interval time-varying delays

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ABSTRACT

In this paper, the problem of stability analysis for static neural networks with interval time-varying delays is considered. By the consideration of new augmented Lyapunov functionals, new and improved delay-dependent stability criteria to guarantee the asymptotic stability of the concerned networks are proposed with the framework of linear matrix inequalities (LMIs), which can be solved easily by standard numerical packages. The enhancement of the feasible region of the proposed criteria is shown via two numerical examples by the comparison of maximum delay bounds.

1. Introduction

When the mathematical model of the plant is unknown or ill-defined, some complexities in the analysis and design of dynamic systems are unavoidable and it has been known that intelligent system theories as a analyzing method of the dynamic systems are more effective in such cases. Unlike conventional methods in control theories, intelligent control theories are based on artificial intelligence rather than on a plant model. One of class of artificial intelligence is neural networks. Their stability analysis is a very important and prerequisite task because the application of neural networks heavily depends on the dynamic behavior of equilibrium points. For this reason, during a few decades, neural networks have been extensively applied in many areas such as reconstruction of moving image, signal processing, the tasks of pattern recognition, associative memories, fixed-point computations, and so on [1]. Moreover, to characterize the dynamical evolution rule of neural networks, according to the use of the neural states or the local field states of neurons, the model of neural networks can be classified into static neural networks or local field networks [1–3].

On the other hand, it is also needed to pay attention to a delay in the time. It is well known that, in the implementation of the networks, there are the finite speed limit of information processing and its attendant time-delay. The delay leads to undesirable dynamic behaviors such as oscillation and instability of the networks. Therefore, the study on stability analysis for various systems with time-delay has been widely investigated [4–15].

Return to static neural network, this network is also extended to the stability problem with time-delay [16–22]. Above all, in [16], the delay-independent and dependent criteria for static neural networks were derived. Zuo et al. [18] investigated

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the problem of delay-dependent stability for time-delay static neural networks by considering some semipositive-definite free matrices. In [19], the stability and dissipativity problems of static neural networks with time-varying delay were investigated by using the delay partitioning technique. Sun and Chen [20] proposed the stability criteria for a class of static neural networks by constructing the new augmented Lyapunov functional which fully uses the information about the lower bound of the delay and contains some new double integral and triple-integral terms. Li et al. [21] developed a unified approach in stability analysis of generalized static neural networks with time-varying delays and linear fractional uncertainties by utilizing some novel transformation and discretized scheme. Here, in order to obtain more tighter lower bounds of integral terms of quadratic form, Wirtinger-based integral inequality in [12] is the recent remarkable tool in delay-dependent stability analysis of dynamic systems with delays. Therefore, there are scopes for further enhanced results in stability analysis of static neural networks with time-delay.

With this motivation mentioned above, in this paper, the problem to get improved delay-dependent stability criteria for the static neural networks with interval time-varying delays are investigated. Here, stability or stabilization of system with interval time-varying delays has been a focused topic of theoretical and practical importance [23] in very recent years. The system with interval time-varying delays means that the lower bounds of time-delay which guarantees the stability of system is not restricted to be zero, and include networked control system as one of typical examples. By construction of a suitable augmented Lyapunov–Krasovskii functional and utilization of Wirtinger-based integral inequality [12] with reciprocally convex approach [13], an improved stability criterion for guaranteeing the asymptotic of static neural networks is derived in Theorem 1 with the framework of LMIs which can be formulated as convex optimization algorithms which are amenable to computer solution [24]. Also, inspired by the works of [13,14], the reciprocally convex approach and zero equality are utilized with some decision variables to reduce the conservatism of the stability criterion. Based on the result of Theorem 1, a further improved result will be proposed in Theorem 2 by introducing a newly augmented Lyapunov–Krasovskii functional. Finally, two numerical examples are included to show the effectiveness of the proposed methods.

Notation Throughout this paper, the used notations are standard. $\mathbb{R}^n$ is the n-dimensional Euclidean vector space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. For symmetric matrices X and Y, $X>Y$ means that the matrix $X-Y$ is positive definite, whereas $X\geqslant Y$ means that the matrix $X-Y$ is nonnegative. $I_n, 0_n$ and $0_{m\cdot n}$ denote $n\times n$ identity matrix, $n\times n$ and $m\times n$ zero matrices, respectively. $\mathtt{diag}\{\ldots\}$ denotes the block diagonal matrix. For square matrix X, $\mathtt{sym}\{X\}$ means the sum of X and its symmetric matrix X^T; i.e., $\mathtt{sym}\{X\}=X+X^T$. $X_{[f(t)]}\in\mathbb{R}^{m\times n}$ means that the elements of matrix $X_{[f(t)]}$ include the scalar value of $f(t)$; i.e., $X_{[f_0]}=X_{[f(t)=f_0]}$.

2. Preliminaries and problem statement

Consider the following static neural networks with time-varying delays:

$$\dot{y}(t)=-Ay(t)+g(Wy(t-h(t))+u(t)),\tag{1}$$

where n denotes the number of neurons in a neural network, $y(t)\in\mathbb{R}^n$ is the neuron state vector, $g(Wy(t))=[g_1(W_1y_1(t)),\ldots,g_n(W_ny_n(t))]^T\in\mathbb{R}^n$ denotes the neuron activation function vector, $u(t)\in\mathbb{R}^n$ is the input vector, $A=diag\{a_1,\ldots,a_n\}\in\mathbb{R}^{n\times n}$ with $a_i>0$ $(i=1,\ldots,n)$ is a positive diagonal matrix, $W=[W_1^T,\ldots,W_n^T]^T\in\mathbb{R}^{n\times n}$ is the delayed interconnection weight matrix. The delay $h(t)$ is time-varying function satisfying

$$h_m\leqslant h(t)\leqslant h_M, \dot{h}(t)\leqslant h_D,$$

where h_m and h_M are known positive scalars, and h_D is a constant.

It is assumed that the neuron activation functions satisfy the following condition.

Assumption 1. The neuron activation functions $g_i(\cdot)$ $(i=1,\ldots,n)$ are continuous, bounded and satisfy

$$k_i^-\leqslant\frac{g_i(u)-g_i(v)}{u-v}\leqslant k_i^+,\ \ \forall u,v\in\mathbb{R},\ \ u\neq v,\ \ g_i(0)=0,\tag{2}$$

where k_i^+ and $k_i^-(k_i^+>k_i^-)$ are constants.

For simplicity, in stability analysis of the system (1), the equilibrium point $y^*=[y_1^*,\ldots,y_n^*]^T$ whose uniqueness has been reported in [25] is shifted to the origin by utilizing the transformation $x(\cdot)=y(\cdot)-y^*$, which leads the system (1) to the following form:

$$\dot{x}(t)=-Ax(t)+f(Wx(t-h(t))),\tag{3}$$

where $x(t)\in\mathbb{R}^n$ is the state vector of the transformed system,

$f(Wx(\cdot))=[f_1(W_1y_1(t)),\ldots,f_n(W_ny_n(t))]^T\in\mathbb{R}^n$ with $f(Wx(\cdot))=g(Wx(\cdot)+y^*)-g(Wy^*+u)$ and $f(0)=0$.

It should be noted that the functions $f_i(\cdot)$ $(i=1,\ldots,n)$ satisfy the following condition:

$$k_i^-\leqslant\frac{f_i(u)-f_i(v)}{u-v}\leqslant k_i^+,\ \ \forall u,v\in\mathbb{R},\ \ u\neq v,\ \ f_i(0)=0.\tag{4}$$

From (4), if $v=0$, then we have