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Adaptive decision dynamics: Bifurcations, multistability and chaos

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ABSTRACT

In this paper we propose a model describing the dynamical process of decision and opinion formation of two economic homogeneous interacting and boundedly rational agents. The decisional process represented in our model is given by an adaptive adjustment mechanism in which two agents take into account the difference between their own opinion and the opinion of the other agent. The smaller that difference, the larger the weight given to the comparison of the opinions. By means of an auxiliary variable describing the distance between the opinions, we obtain a one-dimensional dynamical system for which we investigate, via analytical and numerical tools, the stability of the unique steady state, its bifurcations, as well as the existence of a globally absorbing interval and of chaotic dynamics. We also investigate multistability phenomena, i.e., the presence of coexisting attractors. Finally, we relax the assumption of homogeneity between agents and we show that there is a strong correspondence between the dynamic behaviors in the scenarios with and without homogeneity.

1. Introduction

The traditional models of economic behavior are based on the assumption of rationality, that is, agents make their decisions according to the maximization of an objective function subject to budget constraints. This approach requires that each agent has both a complete knowledge of the objective function and the computational skills to solve an optimization problem. Some authors believe it would be desirable to revise some of the assumptions of perfect rationality, by allowing more bounded types of rational behavior and learning mechanisms [1]. The introduction of this kind of aspects would produce models describing more realistically some features of the decision making and opinion dynamic formation processes.

In this work our contribution concerns an approach that is based on the idea that economic and social decisions are the result of social interactions of boundedly rational agents that interact and learn from each other over time. In our model the trajectories are not the solution to an intertemporal optimization problem, defined by constant preferences and budget constraints, but rather are the outcome of the interaction of homogeneous agents that adapt their decisions according to the other players' choices. Our model conforms with some approaches analyzing the motivations of decision making within disciplines such as sociology, psychology and marketing [2]. In the present paper we assume that there exist emulation and learning from interaction with other individuals. We do not consider what is called intrinsic utility of the decisional process,

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that is, utility derived from the economic and social activities. We take into account only external influences derived from the observation of the others' behavior. We adopt such an approach in order to highlight the role of social interaction as a source of continuous update of the decisional process. Indeed, communication with others often stems from our needs for social interaction and individuals do not always share information for a specific payoff-related purpose, differently for instance from the context of observational learning, where agents observe others' payoff-maximizing decisions and update their own opinions accordingly (see e.g. [3–5]). More generally, according to [6], two kinds of social influence arise: informational social influence, which describes the updating of opinions according to what others said, and normative social influence, that describes the behavior of stating an opinion that fits to the group norm. The models dealing with the former do not need to include utility components, while the models dealing with the latter usually do. See [7] and the references therein for more details on the topic. According to such distinction, our model describes then the informational social influence. Contexts similar to ours have been considered, for example, in [8–10], where authors deal with social learning frameworks in which agents repeatedly take the weighted average of other agents' current opinions in forming their own opinion for the next period, and partly in [7], where also utilities are taken into account.[1] In all such papers sufficient conditions on the weight matrix which allow to reach a consensus are investigated. More precisely, in the seminal paper [8] the weights, representing the mutual influence between agents, are exogenous and fixed over time. In [9] the weights are instead time-varying and may be updated according to the actual opinion profile and the specific time step. In a framework close to the one in [9], in [10] the author finds different sufficient conditions on the weight structure that lead to a consensus and investigates which are the effects of introducing a group of "persistent" agents, that only perform limited interactions, insisting on the initial weights they assign to others or on their own opinions. Finally, in [7] the consequences of a strategic interaction between agents are analyzed: indeed, it is assumed that agents may report their opinion untruthfully, according to their preferences for conformity.

In the present paper we propose a model with two interacting agents, in which each agent weights also the decision or the opinion of the other agent in forming his own new decision or opinion. This process is modeled by an adaptive adjustment mechanism. In particular, similarly to what assumed in [11,12], the weight with which an agent takes into account the difference between his own opinion and the opinion of the other agent decreases with the distance between the two opinions. Such weight is described by a reactivity term, which can take any positive value. We stress however that, differently from [12], we do not assume that if the distance between the two opinions is larger than a given threshold, then there is no interaction and each agent does not change his own opinion anymore. Moreover, unlike in [7–10], where agents take the weighted average of others' current opinions, and thus the weights belong to $[0,1]$, in our adaptive framework the reactivity γ can assume any positive value: however, when it takes values in $[0,1]$, that is, when there is no overreaction to the decision of the other agent, γ may be interpreted as the weight assigned by an agent to the opinion of the other agent, while $1-\gamma$ represents the weight each agent assigns to his own opinion (see (3.1)). In this sense, our model can be considered as more general than those in [7–10], even if we deal with just two agents and we do not admit any form of strategic interaction. Moreover, we stress that, as it is easy to show, when $\gamma \in (0,1)$ is constant, our unique steady state, corresponding to the unanimity scenario, is stable, in agreement with the results in [7–9]. Notice indeed that the strategic interaction behavior in [7] reduces to the naive learning mechanism in [8] when all agents are honest: on the other hand, we stress that the updating mechanism of the influence weights in [10] is compatible neither with the framework in [8] nor with the one described above.

Given our representation of the decisional mechanism, we aim to study the evolution of the opinions over time, that is, understanding whether they converge towards unanimity or if they give rise to other kinds of dynamical behaviors. We stress that the study of cyclical or divergent behaviors is just sketched in [7], while it is missing in [8–10]. In performing our analysis, we introduce an auxiliary variable describing the distance between the two agents' opinions. In such way, we are led to consider a one-dimensional dynamical system with a unique steady state in the origin, corresponding to the unanimity scenario. We find that an excessive reactivity destabilizes the unanimity fixed point through a first period-doubling bifurcation. A further increase in the reactivity parameter destabilizes the period-two cycle that, differently from the classical period-doubling bifurcation scenario, gives rise, through a double pitchfork bifurcation of the second iterate, to two coexisting period-two cycles, that in turn bifurcate giving rise to a sequence of coexisting attractors of the same type until the emergence of chaos.

Among the results described above, we analytically investigate the stability of the unanimity steady state and the flip bifurcation through which it loses stability, as well as the presence of chaotic dynamics. In particular, the presence of chaos is proved via Theorem 1 in [13], showing that our map has period-three orbits, so that all features related to Li–Yorke chaos follow. Moreover, we rigorously prove the existence of a globally absorbing interval.

On the other hand, due to the complexity of the computations involved, we show only numerically the subsequent pitchfork and the route of period-doubling bifurcations of the coexisting periodic attractors leading to two coexisting chaotic attractors, which then merge into a unique attractor, when increasing the reactivity parameter.

Finally, we study what happens when relaxing the assumption of homogeneity between agents. In particular we allow that the reactivities of the two agents may be different and we investigate which of the results obtained under the homogeneity hypothesis are still valid also in the more general context. Our conclusion is that there is a strong correspondence between the dynamic behaviors in the two scenarios.

[1] Indeed, according to those authors, the model in [7] includes both informational and normative social influence elements.