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Hybrid coincidence and common fixed point theorems in Menger probabilistic metric spaces under a strict contractive condition with an application

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ABSTRACT

We prove some coincidence and common fixed point theorems for two hybrid pairs of mappings in Menger spaces satisfying a strict contractive condition. An illustrative example is given to support the genuineness of our extension besides deriving some related results. Then, we establish the corresponding common fixed point theorems in metric spaces. Finally, we utilize our main result to obtain the existence of a common solution for a system of Volterra type integral equations.

1. Introduction and preliminaries

The notion of probabilistic metric space was initiated by Menger [21], but the study of such spaces received an impetus with the pioneering article of Schweizer and Sklar [30]. By now, many authors have already contributed to the theoretical development of probabilistic metric spaces including applications (e.g. [4,5]). Briefly, we recall some cornerstones of this theory. In 1976, Caristi [3] established a fixed point theorem without any continuity requirement which is often referred as Caristi's fixed point theorem, then Zhang et al. [37] proved set-valued Caristi's theorem in probabilistic metric spaces. Chuan [8] extended the Caristi type hybrid fixed point theorem to Menger spaces. Also a generalization of C-contraction for set-valued mappings is essentially due to Pap et al. [26] wherein a fixed point theorem for (Φ, C)-set-valued contraction in Menger space is proved. Thereafter, Hadžić [11,12] obtained some fixed point theorems for set-valued mappings in probabilistic metric spaces (also see [13,14]). In 1995, Pathak et al. [28] proved a common fixed point theorem for a hybrid pair of weakly compatible mappings in complete Menger spaces which in turn generalizes the corresponding results of Hadžić [12]. Hybrid contraction theorems always form an interesting area of research due to their superiority over single valued case. By now, there already exists an extensive literature on fixed point theorems for single valued and set-valued mappings in metric and probabilistic metric spaces under different contractions and applications and for the work of this kind one can be referred to [6,7,11–15,17–20,24,25,27,33,35–38].

The common fixed point theorems for various contraction as well as generalized contraction mappings in Menger spaces were obtained by many mathematicians but we rarely come across fixed point theorems for mappings satisfying strict contractive conditions in Menger spaces. In 2009, Fang and Gao [9] proved common fixed point theorems in Menger spaces

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satisfying strict contractive conditions for a pair of weakly compatible mappings satisfying the property (E.A). Afterward, Ali et al. [2] proved some unique common fixed point results for two pairs of weakly compatible mappings employing the common property (E.A) which improved the corresponding relevant results of Fang and Gao [9] without any requirement on containment amongst the ranges of involved mappings. Finally, Wu et al. [34] established some hybrid-type common fixed point theorems for two pairs of mappings satisfying the common property (E.A) under strict contractive condition.

Inspired by the results of Wu et al. [34], we prove some common fixed point theorems for two hybrid pairs of mappings in Menger spaces satisfying tangential property under a strict contractive condition. Finally, we use fixed point techniques to obtain the existence of a common solution for a system of Volterra type integral equations.

In the following lines, we present some definitions, remarks and lemmas which will be utilized in the sequel. Throughout this paper, unless otherwise specified, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{N}$ denote the set of real numbers, the set of nonnegative real numbers and the set of positive numbers.

Let (X,d) be a metric space. Then, on the lines of Nadler [23], we have:

(1) $CL(X) = \{A : A \text{ is a non-empty closed subset of } X\}$,
(2) $CB(X) = \{A : A \text{ is a non-empty closed and bounded subset of } X\}$,
(3) for non-empty closed and bounded subsets A, B of X and $x \in X$,

$$d(x,A) = \inf\{d(x,a) : a \in A\}$$

and

$$H(A,B) = \max\{\sup\{d(a,B) : a \in A\}, \sup\{d(b,A) : b \in B\}\}.$$

It is well known that $CB(X)$ is a metric space with the distance H which is known as the Hausdorff–Pompeiu metric on $CB(X)$. Moreover, if (X,d) is complete, then $(CB(X),H)$ is also complete. The following terminology is standard.

Definition 1.1 [30]. A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is called a distribution function if it is non-decreasing left continuous with $\inf_{t\in\mathbb{R}} F(t) = 0$ and $\sup_{t\in\mathbb{R}} F(t) = 1$.

We denote by $\Im$ the set of all distribution functions, while $\mathcal{H}$ always denotes the specific distribution function defined by

$$\mathcal{H}(t) = \begin{cases} 0, & \text{if } t \le 0; \\ 1, & \text{if } t > 0. \end{cases}$$

Definition 1.2 [9]. Let $F_1, F_2 \in \Im$. The algebraic sum $F_1 \oplus F_2$ is defined by

$$(F_1 \oplus F_2)(t) = \sup_{t_1+t_2=t} \min\{F_1(t_1), F_2(t_2)\},$$

for all $t \in \mathbb{R}$.

Definition 1.3 [9]. Let f and g be two functions defined on $\mathbb{R}$ with positive values. The notation $f > g$ means that $f(t) \geqslant g(t)$ for all $t \in \mathbb{R}$, and there exists at least one $t_0 \in \mathbb{R}$ such that $f(t_0) > g(t_0)$.

Definition 1.4 [30]. A mapping $\triangle : [0,1] \times [0,1] \to [0,1]$ is called a t-norm if, for all $a,b,c,d \in [0,1]$, we have

(1) $\triangle(a,1) = a$,
(2) $\triangle(a,b) = \triangle(b,a)$,
(3) $\triangle(c,d) \geqslant \triangle(a,b)$ for $c \geqslant a$, $d \geqslant b$,
(4) $\triangle(\triangle(a,b),c) = \triangle(a,\triangle(b,c))$.

Definition 1.5 [30]. Let X be a non-empty set. An ordered pair $(X,\mathcal{F})$ is called a PM-space if $\mathcal{F}$ is a mapping from $X \times X$ into $\Im$ satisfying the following conditions for all $x,y \in X$ and $t \ge 0$ (we denote $\mathcal{F}(x,y)$ by $F_{x,y}$):

(1) $F_{x,y}(t) = \mathcal{H}(t)$ if and only if $x = y$,
(2) $F_{x,y}(t) = F_{y,x}(t)$,
(3) $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$, then $F_{x,z}(t+s) = 1$, for all $z \in X$ and $s \geqslant 0$.