



Travelling waves of iodate-arsenous-acid reaction with a constant electric field



Guirong Liu*, Weiping Yan

School of Mathematical Sciences, Shanxi University, Taiyuan, Shanxi 030006, China

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ABSTRACT

The effects of applying a constant electric field E to an iodate-arsenous-acid reaction of mixed order are considered, where the reacting species, A and B , are present. The resulting equations are studied in detail by obtaining conditions for the existence and nonexistence of travelling waves. By using a novel approach, explicit bounds $c_*(D, E)$ and $c^*(D, E)$ are derived such that there is a unique travelling wave of every speed $c \geq c^*(D, E)$ and there does not exist any travelling wave of speed $c < c_*(D, E)$, where $c_*(D, E)$ and $c^*(D, E)$ are two functions of constant electric field E and D , the ratio of diffusion coefficients of the species A and B . Numerical simulations of the resulting equations confirm our main results.

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1. Introduction

The coupling of a chemical reaction with diffusion and/or ionic migration can, under suitable initiation conditions, give rise to the propagation of a locally applied stimulus in the form of a travelling wave. Such waves play a fundamental role in many chemical and biological systems and a detailed consideration of their properties is a necessary pre-requisite for understanding the various complex processes involved in such systems.

Recently, considerable efforts have been made in studying travelling waves of reaction–diffusion systems. In [1], it has been shown that external noise of appropriate intensity alone is sufficient to evoke coherent travelling waves in a similar reaction–diffusion system. Later, the same has been demonstrated also for internal noise in [2]. It was also shown that travelling waves are robust to periodic forcing in [3], but not robust to rewiring of the regular interaction graph in [4]. It was also shown that delays can enhance the coherence of travelling waves in [5]. Finally, a more pedagogically oriented introduction into noise-driven reaction diffusion systems has been presented in [6].

Systems based on the iodate-arsenous-acid reaction have been shown to exhibit travelling waves and have been extensively studied experimentally, see [7–13]. The subsequent studies [14–18] have shown that applying an electric field to this system alters both the propagation speed of the reaction front and the final outcome of the reaction.

In addition, reaction diffusion systems subject to electric field also emerge naturally in liquid crystals, which has been studied recently in [19,20]. Traveling waves in the form of coherent dynamical states have been reported as well, and it has been shown that an external electric field can significantly improve the coherent dynamics of a liquid crystal cell.

In this paper, we shall study the two-dimensional iodate-arsenous-acid reaction with a constant electric field. The system that governs iodate-arsenous-acid reaction takes the form

* Corresponding author.

E-mail address: lgr5791@sxu.edu.cn (G. Liu).

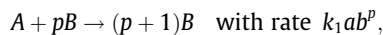
$$\begin{cases} u_x + v_y = 0, \\ w_x = -\frac{\mu}{K}u - g\rho(a, b), \\ w_y = -\frac{\mu}{K}v, \\ a_t + ua_x + va_y + D_A \mathcal{E}a_x = D_A \Delta a - k_1 ab^p - k_2 ab^q, \\ b_t + ub_x + vb_y + D_B \mathcal{E}b_x = D_B \Delta b + k_1 ab^p + k_2 ab^q \end{cases} \quad (1.1)$$

with an equation of state

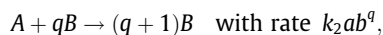
$$\rho(a, b) = \rho_0 + \gamma_1 a + \gamma_2 b,$$

where $\rho_0, \gamma_1, \gamma_2, q > p > 1$ are positive constants. Here w is the pressure, ρ is the density, u, v are the velocity components in the x and y directions, respectively, a and b are the concentrations of two chemical species averaged across the gap and $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator. For the constants, g is the gravitational constant, μ is the viscosity of the field, K is the permeability, D_A and D_B are the diffusion coefficients of reactants A and B , and \mathcal{E} is the electric field strength.

The underlying chemical reaction is the mixture of reaction of the order $p + 1$:



and reaction of the order $q + 1$:



where A and B represent, respectively, IO_3^- and Γ^- with concentrations a and b , and k_1, k_2 are the rate constants.

With the introduction of stream function ψ , elimination of pressure from the equations and dimensionless transformation, we derive

$$\begin{cases} \Delta \psi = -(\gamma_1 a_x + \gamma_2 b_x), \\ a_t + \Gamma(\psi_y a_x - \psi_x a_y) = \Delta a - Ea_x - ab^p - kab^q, \\ b_t + \Gamma(\psi_y b_x - \psi_x b_y) = D \Delta b - DEb_x + ab^p + kab^q, \end{cases} \quad (1.2)$$

where $D = D_B/D_A$, k is proportional to k_2/k_1 , $E = \text{constant} \times \mathcal{E}$, and Γ a parameter proportional to the field-free pressure difference of reactant and product.

Since we are mainly interested in the existence and nonexistence of planar travelling wave, we shall only consider the one-dimensional case with the pressure difference $\Gamma = 0$. In this case, the system (1.2) reduces to the following reaction–diffusion system

$$\begin{cases} a_t = a_{xx} - Ea_x - ab^p - kab^q, \\ b_t = Db_{xx} - DEb_x + ab^p + kab^q, \end{cases} \quad (1.3)$$

where a, b are functions of (x, t) ; $D > 0, E > 0, q > p > 1, k > 0$ are constants. For more details, see [16].

For quadratic autocatalytic reaction ($A + B \rightarrow 2B$), Merkin et al. [15] studied the following reaction–diffusion system

$$\begin{cases} a_t = a_{xx} - Ea_x - ab, \\ b_t = Db_{xx} - DEb_x + ab. \end{cases} \quad (1.4)$$

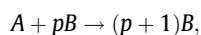
They obtained the conditions for the existence of travelling waves of system (1.4).

For cubic autocatalytic reaction ($A + 2B \rightarrow 3B$), Zdražil et al. [16], and Merkin et al. [17] studied the following reaction–diffusion system

$$\begin{cases} a_t = a_{xx} - Ea_x - ab^2, \\ b_t = Db_{xx} - DEb_x + ab^2. \end{cases} \quad (1.5)$$

In [16], the effects of constant electric fields on the buoyant stability of reaction fronts have been investigated for a range of D and electric field strength E . In [17], they showed the ranges of D and electric field strength E over which travelling waves do not exist.

In [18], Merkin considered a general order for the autocatalytic reaction



where $p \geq 1$. The corresponding reaction–diffusion system takes the form

$$\begin{cases} a_t = a_{xx} - Ea_x - ab^p, \\ b_t = Db_{xx} - DEb_x + ab^p. \end{cases} \quad (1.6)$$

It was shown that, when the ratio D of diffusion coefficients of autocatalyst and substrate is less than some value D_0 (which depends on p), the effect of a negative electric field is to increase the wave speed of travelling waves over its field-free value.

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