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Real eigenvalue bounds of standard and generalized real interval eigenvalue problems

Huinan Leng

Department of Mathematics, East China University of Science and Technology, Shanghai 200237, China

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ABSTRACT

In this paper, we study the real eigenvalue bounds of standard and generalized real interval eigenvalue problems. Based on some known sufficient conditions for the regularity of interval matrices, a new algorithm for computing real eigenvalue bounds for real interval matrices is proposed. It can easily be extended for applying to generalized real interval eigenvalue problems. One advantage of the method is that the computation procedure takes less time than former methods we have proposed. Therefore it can be applied for solving larger interval eigenvalue problems. Finally, some numerical examples are presented which show the effectiveness of the method.

1. Introduction

In the real life and actual computation, parameters are inevitably uncertain due to inaccurate measurement, hypothesis simplification of models, variety of system data and calculation errors, etc. As a result, how to deal with the influence of uncertain parameters to system characteristics is of great importance and has been studied widely in literature.

There are usually two main types of methods to solve problems with uncertainties. One is the type of probabilistic methods [1]. In these methods uncertainties are modeled with random variables or random fields in order to obtain probability density functions of objective variables. The other type uses interval treatments [2], in which uncertainties are described as interval numbers, interval vectors or interval matrices. Then intervals containing all possible solutions need to be computed. In this paper, we study eigenvalues of uncertain matrices by the interval type, then the problems are treated as the interval eigenvalue problems.

Much research work has been done on interval eigenvalue problems since 1980s, among which some were performed on the stability of dynamic interval systems [3] in the control field, since the stability depends on the extremal eigenvalue bounds of certain interval matrices. Besides, some work turned to require all eigenvalue bounds of interval matrices [4–6], which play certain important roles in mechanics and engineering fields, etc. Among all effective contributions to interval eigenvalue problem, Deif [7] developed an effective method for the standard interval eigenvalue problem by using the eigenvalue inequalities and nonlinear programming theorems under the condition that the signs of the components of eigenvectors remain invariable. This method can yield the exact bounds. However, there exists no efficient criterion for judging the required condition in advance [8], so the application of Deif's method is restricted in some sense. In [9], Rohn and Deif pursued a slightly different approach. They investigated the set of real eigenvalues of an interval matrix pertaining to eigenvectors of a given sign pattern. Additionally, an algorithm [10] for calculating exact real eigenvalue bounds of standard interval eigenvalue problems was proposed. Later on this method was extended to generalized interval eigenvalue problems [11]. However, needless to say, the accuracy is always at the cost of large occupation of computation time.

E-mail address: hnleng@ecust.edu.cn

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For some known approximate approaches, Qiu et al. [12] studied a method to produce approximate eigenvalue bounds based on an interval perturbation formula. In [13], Hladik et al. presented an approximation algorithm which could yield inner eigenvalue bounds for symmetric interval matrices. Besides, they also proposed a filtering method [14] that iteratively improved the approximation by giving an outer approximation of the real eigenvalue set of an interval matrix. Considering the exact eigenvalue bounds were not easy to be directly calculable, Matcovschi et al. [15] developed a procedure on the evaluation of right bounds for the eigenvalue ranges of interval matrices. More generally, Kolev [16,17] addressed the study of determining the interval solution of the parametric eigenvalue problem $A(p)x = \lambda x$ for $A(p) \in R^{n\times n}$, where the matrix elements $a_{ij}(p)$ are linear or non-linear continuous functions of the parameter vector p which belongs to an interval vector. In addition, some studies [18,19] dealt with complex eigenvalue bounds of real and complex interval matrices. For instance, based on the idea of reducing the problem to enclosing eigenvalues of symmetric interval matrices, a theorem [19] was proposed to estimate bounds for extremal real and imaginary parts of interval eigenvalues.

In this study, some new algorithms are presented for calculating bounds to real eigenvalues of standard and generalized real interval eigenvalue problems based on several sufficient conditions for regularity of interval matrices. Numerical examples are also computed by applying these algorithms to demonstrate the effectiveness and practicability of the proposed method.

2. Preliminaries

Let $A = [a_{ij}]_{i,j=1}^{n}$, $\overline{A} = [\overline{a_{ij}}]_{i,j=1}^{n}$ and $\underline{A} = [\underline{a_{ij}}]_{i,j=1}^{n}$ be $n \times n$ real matrices. Here $\underline{A} \leqslant A \leqslant \overline{A}$ are in the sense of $\underline{a_{ij}} \leqslant a_{ij} \leqslant \overline{a_{ij}}$ for $i,j = 1,2,\ldots,n$. We define an $n \times n$ real interval matrix A^I as follows, which can be treated as a family of $n \times n$ real matrices:

$$A^I := [\underline{A}, \overline{A}] = \{A \,|\, \underline{A} \leqslant A \leqslant \overline{A}\} = \{[a_{ij}] : \underline{a_{ij}} \leqslant a_{ij} \leqslant \overline{a_{ij}} \text{ for } i,j = 1,2,\ldots,n\}.$$

We denote the midpoint and the radius of A^I respectively by

$A_c := (\overline{A} + \underline{A})/2$ and $\triangle A := (\overline{A} - \underline{A})/2$.

For an interval matrix A^I, the corresponding symmetric interval matrix A^s is defined as a family of all symmetric matrices in A^I, that is, $A^s := \{A \in A^I | A = A^T\}$. Furthermore, the matrices satisfying $\{A | a_{ij} = \underline{a_{ij}} \text{ or } a_{ij} = \overline{a_{ij}}\}$ are said to be the vertex matrices of the interval matrix A^I.

The standard real interval eigenvalue problem is defined as

$$Ax = \lambda x, \quad \text{where } A \in A^I. \tag{1}$$

Here λ is an eigenvalue of the problem (1) and x is the corresponding eigenvector. Let all eigenvalues λ_i for $i = 1,2,\ldots,n$ of the problem (1) be real and arranged in an ascending order.

The exact eigenvalue interval λ^I of (1) is defined by an interval vector

$$\lambda^I = [\underline{\lambda}, \overline{\lambda}] = (\lambda_i^I), \;\; \lambda_i^I = [\underline{\lambda_i}, \overline{\lambda_i}], \quad \text{for } i = 1,2,\ldots,n, \tag{2}$$

in which the interval $[\underline{\lambda_i}, \overline{\lambda_i}]$ is the smallest one that encloses all possible eigenvalues λ_i of the problem (1).

The generalized real interval eigenvalue problem can be similarly defined as

$$Kx = \mu Mx, \quad \text{where } K \in K^I, \; M \in M^I, \tag{3}$$

where K^I and M^I are real interval matrices defined as same as above definition for A^I, and μ is the eigenvalue of the problem (3), x is the corresponding eigenvector. Let all eigenvalues μ_i for $i = 1,2,\ldots,n$ of the problem (3) be real and arranged in an ascending order.

The exact eigenvalue interval μ^I of (3) is defined by an interval vector

$$\mu^I = [\underline{\mu}, \overline{\mu}] = (\mu_i^I), \quad \mu_i^I = [\underline{\mu_i}, \overline{\mu_i}], \quad \text{for } i = 1,2,\ldots,n, \tag{4}$$

which has the same meaning described for the problem (1).

Among all studies for computing eigenvalue bounds of the problems (1) and (3), most methods obtain outer bounds for exact eigenvalue intervals defined in (2) and (4). Only a few obtain the exact results since it is a difficult task. The awkwardness is either the conditions required are difficult to be verified or a large amount of calculation is needed as the order of matrices increases.

Now we denote the outer eigenvalue interval for the standard real interval eigenvalue problem (1) by

$$\alpha^I = [\underline{\alpha}, \overline{\alpha}] = (\alpha_i^I), \quad \alpha_i^I = [\underline{\alpha_i}, \overline{\alpha_i}], \quad \text{for } i = 1,2,\ldots,n, \tag{5}$$

such that $\lambda^I \subset \alpha^I$, i.e. $\lambda_i^I \subset \alpha_i^I$, and the inner eigenvalue interval by

$$\xi^I = [\underline{\xi}, \overline{\xi}] = (\xi_i^I), \quad \xi_i^I = [\underline{\xi_i}, \overline{\xi_i}], \quad \text{for } i = 1,2,\ldots,n, \tag{6}$$

such that $\xi^I \subset \lambda^I$, i.e. $\xi_i^I \subset \lambda_i^I$.

Similarly, denote the outer eigenvalue interval for the generalized real interval eigenvalue problem (3) by