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Survival probabilities in a discrete semi-Markov risk model

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ABSTRACT

In this paper, we consider the survival probability for a discrete semi-Markov risk model, which assumes individual claims are influenced by a Markov chain with finite state space and there is autocorrelation among consecutive claim sizes. Our semi-Markov risk model is similar to the one studied in Reinhard and Snoussi (2001,2002) [1,2] without the restriction imposed on the distributions of the claims. In particular, the model of study includes several existing risk models such as the compound binomial model (with time-correlated claims) and the compound Markov binomial model (with time-correlated claims) as special cases. The main purpose of the paper is to develop a recursive method for computing the survival probability in the two-state model, and present some numerical examples to illustrate the application of our results.

1. Introduction

Markov-modulated risk models, where the surplus processes are influenced by an environmental Markov chain, has attracted a lot of attention recently. Some recent papers on ruin problems for these models include [3–10] and references therein.

In a Markov-modulated risk model, the premiums, claim amounts and claim number process are usually assumed to be (conditionally) independent given the environmental Markov chain, that is, they only depend on the current state of the Markov chain. However, this (conditional) independence assumption may be somewhat too strong in some applications. Janssen and Reinhard [11] first considered a semi-Markovian dependence structure where the claim amounts and inter-claim times not only depend on the current state but also the next state of the environmental Markov chain. They derived the survival probabilities in terms of an infinite series of matrix convolutions. Albrecher and Boxma [12] considerably generalized the approach of Janssen and Reinhard [11] and investigated the discounted penalty function in such a risk model by means of Laplace-Stieltjes transforms. Recently, Cheung and Landriault [13] further studied the work of Albrecher and Boxma [12] by relaxing some assumptions pertaining to the inter-claim time distribution.

With a strict restriction imposed on the total claim amount, Reinhard and Snoussi [1,2] considered a discrete-time semi-Markov risk model and derived recursive formulae for calculating the distribution of the surplus just prior to ruin and the distribution of the deficit at ruin in a special case. In this paper, we shall relax the restriction of Reinhard and Snoussi [1,2] and derive closed-form expressions for the ruin probability in the two-state semi-Markov risk model. Since the model of study embraces some existing discrete-time risk models including the compound binomial model (with time-correlated

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claims) and the compound Markov binomial model (with time-correlated claims), the present paper generalizes the study of ruin probability for these risk models.

The rest of the paper is organized as follows. In Section 2, we present the mathematical formulation of the discrete semi-Markov model. In Section 3, we derive recursive formulae for computing survival probabilities for the model. Section 4 is devoted to finding the initial values for applying the recursive formulae. Several special cases of our model are considered in Section 5. Finally, some numerical examples are presented in Section 6.

2. The model

The model considered in this paper is based on a discrete-time semi-Markov risk model proposed by Reinhard and Snoussi [1,2]. Let $(J_n, n \in \mathbb{N})$ be a homogeneous, irreducible and aperiodic Markov chain with finite state space $M = \{1, \ldots, m\}$ $(1 \leqslant m < \infty)$. Its one-step transition probability matrix is given by

$$\mathbf{P} = (p_{ij})_{i,j \in M}, \quad p_{ij} = \mathbb{P}(J_n = j | J_{n-1} = i, J_k, k \leqslant n-1),$$

with a unique stationary distribution $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_m)$. The insurer's surplus at the end of the t-th period $(t \in \mathbb{N}_+)$, U_t, has the form

$$U_t = u + ct - \sum_{i=1}^{t} Y_i, \quad t \in \mathbb{N}_+, \tag{1}$$

where Y_i denotes the total claim amount in the i-th period, and $c \in \mathbb{N}_+ = \{1, 2, \ldots\}$ is the amount of premium per period. We further assume that the insurer has a non-negative initial surplus u, and that Y_t's are nonnegative integer-valued random variables. The distribution of Y_t's is influenced by the environmental Markov chain $(J_n, n \in \mathbb{N})$ in the way that (J_t, Y_t) depends on $\{J_k, Y_k; k \leqslant t-1\}$ only through J_{t-1}. Define

$$g_{ij}(l) = \mathbb{P}(Y_t = l, J_t = j | J_{t-1} = i, J_k, Y_k, k \leqslant t-1), \quad l \in \mathbb{N},$$

which describes the conditional joint distribution of Y_t and J_t given the previous state J_{t-1}, and plays a key role in the following derivations. Note that the variables $\{Y_t, t \in \mathbb{N}_+\}$ are conditionally independent given the environmental Markov chain.

Assume that premiums are received at the beginning of each time period with $c = 1$. As was mentioned in [14], this is the case when claim amounts are multiples of the periodic premium; and hence, after a change of monetary units, the periodic premium is 1. Assume further that for all i and j,

$$\mu_{ij} = \sum_{k=0}^{\infty} k g_{ij}(k) < \infty,$$

and define

$$\mu_i = \sum_{j=1}^{m} \mu_{ij}, \quad i \in M.$$

Define $\tau = \inf\{t \in \mathbb{N}_+ : U_t < 0\}$ as the time of ruin and let

$$\psi_i(u) = \mathbb{P}(\tau < \infty | U_0 = u, J_0 = i), \quad i \in M, \ u \in \mathbb{N}$$

be the ultimate ruin probability given the initial surplus u and the initial environment state i. Let $\phi_i(u) = 1 - \psi_i(u)$ be the corresponding survival probability. To make sure that ruin is not certain, we assume that the positive safety loading condition holds, that is, $\sum_{i=1}^{m} \pi_i \mu_i < 1$. Here our aim is to derive recursive formulae for computing $\phi_i(u)$.

Remark 1. In [1,2], it is assumed that

$$\begin{cases} g_{ij}(0) = 0, & \forall j \in M, \ i \neq 1, \\ \sum_{j \in M} g_{1j}(0) > 0. \end{cases} \tag{2}$$

This condition says that zero claims are only possible when the state prior to the occurrence of the claim is state 1. Without this condition, the recursive formulae cannot be established using their method even for $m = 2$. Here we relax the condition, and consider the case of $m = 2$. Furthermore, without restriction (2), model (1) covers the compound binomial model (with time-correlated claims) and the compound Markov binomial model (with time-correlated claims) as special cases. For details, see Section 5.

3. Recursive formulae

In this section, we derive recursive formulae for computing survival probabilities $\phi_i(u), i = 1, 2$. To do this, we adopt the method of [15] in which a dividend problem for the same discrete semi-Markov risk model was considered.