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Adaptive chatter free sliding mode control for a class of uncertain chaotic systems

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ABSTRACT

The chaos control for a class of uncertain chaotic system is studied. By using the Lyapunov stability theory and adaptive control technique, an adaptive chatter free sliding mode control is proposed without the knowledge of the bounds of the uncertain term representing the model uncertainty and unknown disturbance. Numerical simulation result is provided to show the effectiveness of the proposed control law.

1. Introduction

Chaotic behavior is an interesting nonlinear phenomenon and has been received more and more attentions in the last decades due to its powerful applications in chemical reactions, power converters, information processing, secure communication, and so on [1].

The problem of chaos control has widely been studied since the early 1990s [2]. Nowadays, several control methods have been proposed to deal with this problem, such as feedback linearization [3], optimal control [4,5], adaptive control [6,7], backstepping control [8,9], sliding mode control (SMC) [10,11], neural networks method [12–14], delay stochastic method [15], Gershgorin circle method [16]. Among the existing control methodologies, the sliding mode control approach [17] is a powerful and robust tool to deal with uncertain chaotic systems and has been successfully applied in controlling chaos [18]. However, most conventional sliding mode control laws suffer from the undesirable chattering phenomenon, which is caused by the discontinuous nature of the sign function in control input.

Recently, in order to eliminate the undesirable chattering, a chatter free sliding mode control strategy was proposed in [19] under the assumption that the upper bounds of the uncertain terms and their first derivatives are known. However, the same problem has not been studied when the upper bounds of the uncertain terms and their first derivatives are unknown. Without that assumption, the controller in [19] does not work.

In this paper, under the assumption that the upper bounds of the uncertain terms and their first derivatives are unknown, we aim to present a chatter free sliding mode control strategy for a class of uncertain chaotic systems, which is more general and includes the chaotic systems considered in [19] as a special case. To overcome the difficulty caused by the unknown upper bounds, we have to combine the sliding mode control method with the adaptive control technique to deal with our problem.

The organization of this paper is as follows. In Section 2, the system dynamics of the uncertain chaotic systems is described and the problem is formulated. In Section 3, the design procedure for the adaptive chatter free sliding mode controller is proposed. A simulation study is presented to demonstrate the effectiveness of the proposed method in Section 4. Finally, conclusion is presented in Section 5.

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2. System description and problem formulation

Consider a class of uncertain chaotic systems in the following form:

$$\dot{x} = f(t,x) + d(t,x) + u(t), \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in \mathbb{R}^n$ is the state vector, $f(t,x) : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ denotes a continuous nonlinear vector field, $d(t,x) : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ is the uncertain term representing the model uncertainty and unknown disturbance, and $u(t) \in \mathbb{R}^n$ is the control input.

The control objective of this paper is to design an adaptive chatter free sliding mode control law, in the presence of the model uncertainty and unknown disturbance, such that the state of closed-loop system is asymptotically stable for any given initial conditions, i.e.,

$$\lim_{t \to +\infty} \|x(t)\| = 0. \tag{2}$$

Remark 2.1. It is easy to see that system (1) reduces to the system (1) in [19] if $f(t,x) = Ax + F(t,x)$ and $d(t,x) = d(t)$. However, we allow the uncertain term depends on the state variable x, which is different from the uncertain term $d(t)$ in [19].

3. Adaptive chatter free SMC design and stability analysis

In order to present an adaptive chatter free sliding mode control law, as in [19], we introduce the following dynamical sliding mode surfaces:

$$\sigma_i(t) = \dot{s}_i(t) + \lambda_i s_i(t), \quad i = 1, 2, \ldots, n, \tag{3}$$

where $\lambda_i, i = 1, 2, \cdots, n$, are positive constants, and s_i is defined as following:

$$s_i(t) = k_i \int_0^t x_i(\tau) d\tau + x_i(t), \quad i = 1, 2, \ldots, n, \tag{4}$$

where $k_i, i = 1, 2, \cdots, n$, are positive constants.

Before proceeding further, we make the following assumption.

Assumption 3.1. *There exist unknown positive constants $c_i, i = 1, 2, \ldots, n$, such that*

$$\left| \sum_{j=1}^{n} \frac{\partial d_i(t,x)}{\partial x_j} \dot{x}_j + \frac{\partial d_i(t,x)}{\partial t} \right| + (\lambda_i + k_i)|d_i(t,x)| \leqslant c_i, \quad i = 1, 2, \ldots, n. \tag{5}$$

Remark 3.2. Note that, unlike [19], we allow the upper bound parameter c_i is unknown.

Now, we are ready to state our main result.

Theorem 3.3. *Under Assumption 3.1, let the adaptive sliding mode controller be*

$$\begin{aligned} \dot{u}_i &= -(\lambda_i + k_i)(u_i + f_i(t,x)) - \sum_{j=1}^{n} \frac{\partial f_i(t,x)}{\partial x_j} \dot{x}_j - \frac{\partial f_i(t,x)}{\partial t} - \lambda_i k_i x_i - \epsilon_i \hat{c}_i sign(\sigma_i), \\ \dot{\hat{c}}_i &= \epsilon_i |\sigma_i|, \end{aligned} \tag{6}$$

where $\hat{c}_i$ is the estimation of the unknown upper bound $c_i, i = 1, 2, \ldots, n$, and $\epsilon_i > 1, i = 1, 2, \ldots, n$ are some constant parameters which will be specified by the designer.

Then, for any given initial condition, the state $x(t)$ of closed-loop system is asymptotically stable.

Proof. Firstly, consider the following Lyapunov function candidate

$$V = \frac{1}{2} \sum_{i=1}^{n} [\sigma_i^2 + (\hat{c}_i - c_i)^2], \tag{7}$$

then, the derivative of V with respect to time t can be given by

$$\dot{V} = \sum_{i=1}^{n} [\sigma_i \dot{\sigma}_i + (\hat{c}_i - c_i)\dot{\hat{c}}_i]. \tag{8}$$

Combining (8) with (3) and (4), we have