Applied Mathematics and Computation 232 (2014) 595–605

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Couple-group consensus for second-order multi-agent systems with fixed and stochastic switching topologies

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ARTICLE INFO

Keywords:
Couple-group consensus
Multi-agent system
Fixed topology
Stochastic switching topology

ABSTRACT

This paper deals with the couple-group consensus problem for second-order discrete-time multi-agent systems. Both the fixed topology case and the stochastic switching topology case are considered. The couple-group consensus problem is converted into the stability problem of the error system by a linear transformation. For the fixed topology case, we obtain two different conditions of couple-group consensus. For the stochastic switching topology case, we obtain a necessary and sufficient condition of mean-square couple-group consensus. Algorithms are provided to design the allowable control gains. Finally, simulation examples are given to show the usefulness of the theoretical results.

1. Introduction

In the past decade, distributed cooperative control has attracted a tremendous amount of interest. As one important branch of distributed cooperative control, the consensus problem has made a great progress [1–5]. In the early literature, attentions were mainly focus on the single-integrator [6,7]. In [6], the consensus seeking problem of multi-agent systems with dynamically changing interaction topologies was studied, where both discrete and continuous consensus algorithms were considered. The LMI method was used to deal with the robust $\mathcal{H}_\infty$ consensus problem of single-order multi-agent systems with uncertainty in [7]. The consensus problem for double-integrator multi-agent systems has also attracted a lot of attentions since the theoretical framework of consensus problem for multi-agent systems was posed [8,9]. In [8], some necessary and sufficient conditions of consensus for second-order multi-agent systems were obtained, where the systems with communication delays also were considered. In [9], the second-order consensus problem of multi-agent systems with inherent delayed nonlinear dynamics and intermittent communications was studied.

Recently, the multi-agent systems with fixed topology have been extended to switching topology scenario [10,11]. In [10], second-order consensus problem of multi-agent systems with switching topology and communication delay was studied, where the switching signal was arbitrary and the every topology was connected. In [11], the authors studied the finite-time consensus problem for second-order networked multi-agent systems with an undirected switching graph. However, the switching topologies are in the deterministic framework in the above literature. Some consensus results for multi-agent systems with stochastic switching topologies have been obtained [12–14]. In [12], the authors studied the mean square consensusability problem for a network of double-integrator agents with Markov switching topologies. While in [14], an observer-based control strategy for networked multi-agent systems with communication delay and stochastic switching

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http://dx.doi.org/10.1016/j.amc.2014.01.018
0096-3003/

topology was presented. The mean-square consensus problem of multi-agent systems was converted into the mean-square stability problem of an equivalent system by a system transformation.

Very recently, increasing interest has turned to group consensus problems for multi-agent systems. The main idea of group consensus is that the interaction topology includes several subgroups, and each subgroup has a spanning tree. The agents belong to different subgroups will reach different consensus states [15]. In [15], the definition of group consensus was presented and a novel consensus protocol was designed to solve the group consensus problem. In [16], the group consensus problem of multi-agent systems with switching topologies was studied. The group consensus was proved to be equivalent to the asymptotical stability of a class of switched linear systems by a double-tree-form transformation. In [17], another consensus protocol was designed to solve the couple-group consensus problem of multi-agent systems with directed fixed topology. In [18], two different kinds of consensus protocols were given to deal with the group consensus problem for double-integrator dynamic multi-agent systems. In [19], the sampled-data control method was employed to deal with the group consensus problem for multi-agent systems, where the interaction topology is undirected. However, to the best of authors knowledge, the group consensus problems for discrete-time multi-agent systems and the multi-agent systems with stochastic switching topologies have not been fully studied.

In this paper, we study the couple-group consensus problems for second-order discrete-time multi-agent systems with the fixed topology and the stochastic switching topology, respectively. The stochastic switching topologies are assumed to be governed by a homogeneous Markov chain. For the fixed topology case, we obtain a sufficient condition for couple-group consensus based on the eigenvalues of the matrix and a necessary and sufficient condition for couple-group consensus in form of matrix inequality, respectively. For the stochastic switching topology case, we obtain a necessary and sufficient condition for mean-square couple-group consensus in form of matrix inequality. Algorithms are given to design the feasible control gains.

Notation. Let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ represent, respectively, the complex number set, the real number set and the nonnegative integer set. Denote the spectral radius of the matrix M by $\rho(M)$. Suppose that $A, B \in \mathbb{R}^{p\times p}$. Let $A \succeq B$ (respectively, $A \succ B$) denote that $A - B$ is symmetric positive semi-definite (respectively, symmetric positive definite). Given $X(k) \in \mathbb{R}^p$, define $\|X(k)\|_E \triangleq \|E[X(k)X^T(k)]\|_2$, where $E[\cdot]$ is the mathematical expectation. I_n denotes the $n \times n$ identity matrix. $Re(\cdot)$ and $Im(\cdot)$ represent, respectively, the real part and imaginary part of a number. Let $\mathbf{0}$ denote the zero matrix with appropriate dimensions.

2. Graph theory notations

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a directed graph of order n, where $\mathcal{V} = \{v_1, \ldots, v_n\}$ and $\mathcal{E}$ represent the node set and the edge set, respectively. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the adjacency matrix associated with $\mathcal{G}$, where $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. An edge $(v_i, v_j) \in \mathcal{E}$ if agent j can obtain the information from agent i. We say agent i is a neighbor of agent j. Let $N_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$ denote the neighbor set of agent i. The (nonsymmetrical) Laplacian matrix $\mathcal{L}$ associated with $\mathcal{A}$ and hence $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$, where $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}, \forall i \neq j$. A directed path is a sequence of edges in a directed graph in the form of $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots$, where $v_{i_k} \in \mathcal{V}$. A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. A directed spanning tree of $\mathcal{G}$ is a directed tree that contains all nodes of $\mathcal{G}$. A directed graph has or contains a directed spanning tree if there exists a directed spanning tree as a subset of the directed graph, that is, there exists at least one node having a directed path to all of the other nodes. The union of graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is the graph $\mathcal{G}_1 \bigcup \mathcal{G}_2$ with vertex set $\mathcal{V}(\mathcal{G}_1) \bigcup \mathcal{V}(\mathcal{G}_2)$ and edge set $\mathcal{E}(\mathcal{G}_1) \bigcup \mathcal{E}(\mathcal{G}_2)$.

3. Problem formulation and main results

Suppose that the multi-agent system considered consists of $n + m$ agents. Without loss of generality, we assume that the first n agents achieve a consistent state while the last m agents achieve another consistent state. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ denote the topology of multi-agent system considered. Denote $\mathcal{I}_1 = \{1, 2, \ldots, n\}, \mathcal{I}_2 = \{n+1, n+2, \ldots, n+m\}$. Let $\mathcal{V}_1 = \{v_1, v_2, \ldots, v_n\}$ and $\mathcal{V}_2 = \{v_{n+1}, v_{n+2}, \ldots, v_{n+m}\}$ represent the first n agents and the last m agents, respectively. Then, $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2, \mathcal{V}_1 \cap \mathcal{V}_2 = \Phi$. In addition, let $N_{1i} = \{v_j \in \mathcal{V}_1 : (v_i, v_j) \in \mathcal{E}\}$ and $N_{2i} = \{v_j \in \mathcal{V}_2 : (v_i, v_j) \in \mathcal{E}\}$. It is obvious that $N_i = N_{1i} \cup N_{2i}$.

In this paper, we consider the second-order discrete-time multi-agent systems as follows:

$$\begin{cases} \xi_i[k+1] = \xi_i[k] + \zeta_i[k] \\ \zeta_i[k+1] = \zeta_i[k] + u_i[k] \end{cases}, \quad i = 1, 2, \ldots, n+m, \tag{1}$$

where $\xi_i[k],\ \zeta_i[k] \in \mathbb{R}$ are the position and the velocity of the ith agent, respectively; $u_i[k] \in \mathbb{R}$ is the control input of agent i at time k.

3.1. Fixed topology case

In [15–18], the continuous-time consensus algorithms were proposed. Motivated by these results, we consider the following discrete-time consensus algorithm

$$u_i[k] = \begin{cases} \sum_{j\in N_{1i}} a_{ij}[\alpha(\xi_j[k] - \xi_i[k]) + \beta(\zeta_j[k] - \zeta_i[k])] + \sum_{j\in N_{2i}} a_{ij}(\alpha\xi_j[k] + \beta\zeta_j[k]) & \forall i \in \mathcal{I}_1, \\ \sum_{j\in N_{2i}} a_{ij}[\alpha(\xi_j[k] - \xi_i[k]) + \beta(\zeta_j[k] - \zeta_i[k])] + \sum_{j\in N_{1i}} a_{ij}(\alpha\xi_j[k] + \beta\zeta_j[k]) & \forall i \in \mathcal{I}_2, \end{cases} \tag{2}$$