Applied Mathematics and Computation 232 (2014) 632–642

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Discrete and continuous dynamics in nonlinear monopolies ☆

Akio Matsumoto [a,*], Ferenc Szidarovszky [b]

[a] Department of Economics, Chuo University, 742-1 Higashi-Nakano, Hachioji, Tokyo 192-0393, Japan
[b] Department of Applied Mathematics, University of Pécs, Ifúság, u. 6, H-7624 Pécs, Hungary

ARTICLE INFO

Keywords:
Time delay
Inertia
Gradient dynamics
Continuous and discrete dynamics
Bounded rationality
Monopoly

ABSTRACT

Dynamic monopolies are investigated with discrete and continuous time scales by assuming general forms of the price and cost functions. The existence of the unique profit maximizing output level is proved. The discrete model is then constructed with gradient adjustment. It is shown that the steady state is locally asymptotically stable if the speed of adjustment is small enough and it goes to chaos through period-doubling cascade as the speed becomes larger. The non-negativity condition that prevents time trajectories from being negative is derived. The discrete model is converted into the continuous model augmented with time delay and inertia. It is then demonstrated that stability can be switched to instability and complex dynamics emerges as the length of the delay increases and that instability can be switched to stability as the inertia coefficient becomes larger. Therefore the delay has the destabilizing effect while the inertia has the stabilizing effect.

1. Introduction

The literature on monopolies and oligopolies plays a central role in mathematical economics. The existence and uniqueness of the equilibrium were the research issues in early stage and then dynamic extensions become the main topic of researchers. Linear models were first examined, where local asymptotical stability implies global stability. Each model is based on a particular output adjustment scheme. In applying best response dynamics, global information is needed about the profit function while in the case of gradient adjustments only local information is needed to assess the marginal profit. The early results on static and dynamic oligopolies are summarized in [6] and their multiproduct generalization are discussed in [7]. During the last two decades an increasing attention has been given to nonlinear dynamics. Bischi et al. [3] gives a comprehensive summary of the newer development. As a tractable case many authors have examined dynamic monopolies. Baumol and Quandt [2] have investigated cost-free monopolies, the dynamic extensions of which were examined in both discrete and continuous time scales. Their adjustment scheme resulted in convergent processes. Puu [8] assumed cubic price and linear cost functions and considered only discrete time scales. Naimzada and Ricchiuti [5] assumed cubic price and linear cost functions, and their model was generalized by Askar [1] with a general from of the price function keeping the linearity of the cost function. It is shown in these studies that chaotic dynamics arises via period doubling bifurcation.

☆ The authors highly appreciate the financial supports from the MEXT-Supported Program for the Strategic Research Foundation at Private Universities 2013–2017, the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research (C) 24530201 and 25380238) and Chuo University (Grant for Special Research). The usual disclaimers apply.

* Corresponding author.
E-mail addresses: akiom@tamacc.chuo-u.ac.jp (A. Matsumoto), szidarka@gmail.com (F. Szidarovszky).

http://dx.doi.org/10.1016/j.amc.2014.01.101
0096-3003/

In this paper, we reconsider monopoly dynamics in three different points of view. First, we generalize the model of [1] by introducing more general types of the cost function and study local and global stability with the non-negativity condition that prevents time-trajectories from being negative. The discrete-time model is converted into the continuous-time model by introducing a time delay and an inertia in the direction of output change. Second, after examining the stability with continuous time scale, we reveal that the discrete model is less stable than the continuous model. Lastly, we numerically and analytically show that the continuous model gives rise to complex dynamics involving chaos due to delay and inertia.

The paper is organized as follows. In Section 2, the discrete-time model is presented, conditions are derived for local asymptotic stability. In Section 3, the continuous-time model is constructed from the discrete-time model. Stability with respect to time delay and inertia is considered. In the final section, concluding remarks are given.

2. Discrete time model

In this section we construct a discrete time dynamic model of a monopoly, after determining the profit maximizing output. Consider a monopoly, where q is its output, $p(q) = a - bq^{\alpha}$ the price function $(a, b > 0,\ \alpha \geqslant 1)$ and $C(q) = cq^{\beta}$ $(c > 0,\ \beta \geqslant 2)$ its cost function. The profit of the monopoly is given as

$$\pi(q) = (a - bq^{\alpha})q - cq^{\beta}.$$

By differentiation

$$\pi'(q) = a - b(\alpha + 1)q^{\alpha} - c\beta q^{\beta-1} \tag{1}$$

and

$$\pi''(q) = -b\alpha(\alpha + 1)q^{\alpha-1} - c\beta(\beta - 1)q^{\beta-2} < 0.$$

So $\pi(q)$ is strictly concave in q, and since $\pi'(0) = a > 0$ and $\lim_{q\to\infty}\pi'(q) = -\infty$, there is a unique positive profit maximizing output, $\bar{q}$, which is the solution of the first-order condition

$$b(\alpha + 1)q^{\alpha} + c\beta q^{\beta-1} = a. \tag{2}$$

The left hand side is strictly increasing, its value is 0 at $q = 0$ and converges to infinity as $q \to \infty$, therefore the unique positive solution can be obtained by simple computer methods (see, for example, [9]. In the numerical considerations to be done below, we always use the following specification of the parameters,

$$a = 4, \quad b = 3/5 \quad \text{and} \quad c = 1/2,$$

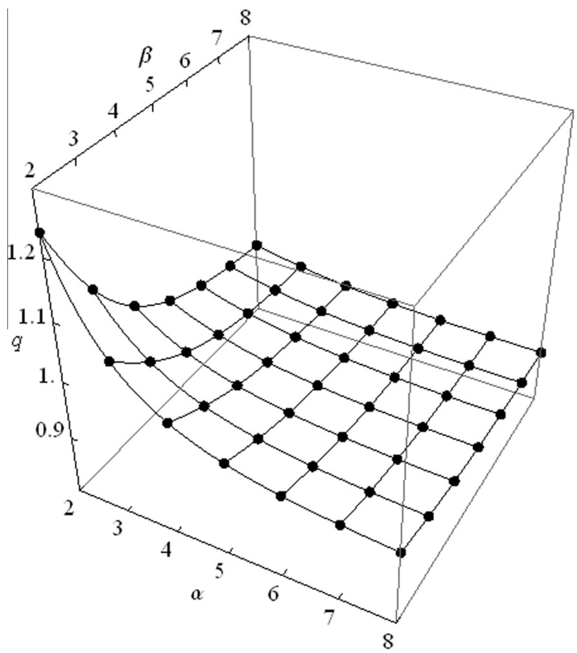

Fig. 1. Determination of the optimal q.