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Insurance pricing using H_∞-control

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ABSTRACT

The paper considers a typical insurance system "suffering" from the three standard "curses": (a) the stochastic nature of claims, (b) the inherent delays in claims settlement and reserving process and (c) the uncertainty concept that endows many of its parameters and especially the investment process. We construct a general multidimensional model for pricing simultaneously one, two or more different insurance products. The responsible decision maker uses the incomplete information of claims and aims to balance the system by the means of a feedback mechanism. The robust stabilization controller of the system is obtained by the means of H_∞-control using typical linear matrix inequalities. Finally, a numerical application is fully investigated providing further insight into the practical problem of pricing assuming the simplest case of a portfolio with a single product.

1. Introduction

The pricing process is one of the basic concerns of an insurance company. The traditional static point of view has already been replaced by the dynamic view using the tools of stochastic control theory. Actually, typical insurance systems have been described by the means of stochastic differential or difference equations while the pricing process has been established as an optimization problem under a certain objective function. In general the "insurance pricing puzzle" is quite complicated and difficult. There are several issues and modelling technicalities such as: the stochastic nature of the parameters, the inherent delay of the available information, the uncertainty of the economic environment and many others. An insurance company starting with an initial capital it continuously receives premiums while pays claims at specific time points and always targets to a positive (but not huge) balance in any future time.

Past research papers have investigated the pricing problem of an insurance system allowing for stochastic variables or delays or other technicalities. Here, we mention only a few of these research efforts as an indicative guide to the huge problem of pricing. Martin-Löf [6] was amongst the pioneers for using the tools of control theory in the insurance pricing context. Vandebroek and Dhaene [10] apply the control theory techniques in the non-life insurance business. Norberg [7], using the standard Brownian motion as the basic tool for modelling, formulates the problem of pricing and reserving within a continuous time framework. His results coincide with the typical results of the traditional approach from risk theory. Zimbidis and Haberman [13] examine the combined effect of delay and feedback on the pricing process. Emms et al. [4] explore the optimal premium strategies within a competitive insurance market using the tools of a standard Brownian motion. Young [12] investigates the pricing of life contracts when mortality is modelled stochastically via the instantaneous Sharpe ratio. Zimbidis [14] further develops the concept of competitive insurance markets using the fractional Brownian motion as the modelling tool for the claim process. Finally, we provide an interesting reference for the modeling of uncertainty in the financial setting. Denis and Martini [3] introduce the uncertainty concept in the financial pricing of a contingent claim. They

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incorporate uncertainty via a family of martingale measures and calculate the superreplication price of a European contingent claim.

In this paper, we investigate the pricing problem considering all the three complications mentioned before: Stochastic nature, delay and uncertainty. The new modelling concept is the uncertainty. We solve the complications by using the tools of H_∞-control. Additionally, we make an allowance for pricing a mixed portfolio of different insurance products.

We start from the very basics, where we consider an insurance company with an initial reserve, an incoming premium flow plus incoming investment proceeds while also an additional outcome of claim payments. The pricing rule (premium determination) is based on past claim experience and over an initial balancing feedback mechanism evaluating the total accumulated surplus (refunding a certain portion of surplus back to the policyholders by reducing the initial level of premium). Now, although the typical description mentioned above indicates premium as input and claims as output variables, in the typical system design we should consider (the vice versa situation) claims as input variable while premiums as output variable. Then, our target is to simultaneously stabilize the premium level charged to the policyholders while also stabilize the state variable: the accumulated surplus fund of the insurance company. The stabilization may be obtained by the attachment of an additional feedback mechanism. That may be described with equations and Fig. 1 below.

[Surplus Fund at time t] = [Initial Fund of the Company at time 0] + [Investment Proceeds up to time t] + [Premiums received up to time t] − [Claims paid up to time t].

[Premium received at time t] = [Average of paid claims per time-unit up to time t] − [% of the Surplus Fund at time t].

The paper is organised as follows: Section 2 contains a short introductory guide to (H infinity) H_∞-control and linear matrix inequalities (LMI's). Section 3 describes the basic structure for the pricing process and the set of required assumptions. Furthermore, we analyze the problem in the multidimensional case allowing for pricing two or more different insurance products. Section 4 presents the theoretical solution of the model for the general multidimensional case while provides a detailed numerical application and practical considerations as regards the insurance pricing for the simplest case of a portfolio with a single product. Finally, Section 5 concludes the paper.

2. H_∞-control and linear matrix inequalities

Control theory and especially optimal control theory has played an important role in many scientific areas and practical problems over the past decades. The last two or three decades, control theory has been fully applied to actuarial problems. A new direction of research for control theory the very last years is the H_∞-control. Actually, H_∞-control is optimal control design when considering the worst exogenous input for a closed loop system. So, H_∞-control offers an ideal framework to investigate problems under uncertain (but somehow bounded) parameters and conditions. Below, we provide a short note on the relevant theory as regards the H_∞-control (see [2] for more details) and linear matrix inequalities (LMIs) (see [1] for more details) that is the powerful tool for solving the respective stability problems. Before going further, we formalize the notation for the matrices i.e. Let A represents a matrix, then A $\succeq\ (\geqslant, \preceq \leqslant,)\,0$ denotes that A is symmetric positive definite (symmetric positive semi-definite, symmetric negative definite, symmetric negative semi-definite).

2.1. H_∞-control (or H infinity control)

We assume an uncertain linear stochastic delayed and controlled differential system,

$$dx(t) = (A(t)x(t) + A_d(t)x(t-\tau(t)) + B(t)u(t) + B_\upsilon\upsilon(t))dt + (\mathrm{E}(t)x(t) + \mathrm{E}_d(t)x(t-\tau(t)) + \mathrm{E}_\upsilon\upsilon(t))dW(t) \quad t \geqslant 0, \tag{1}$$

$$z(t) = Cx(t) + C_d x(t-\tau(t)) + Du(t) \quad t \geqslant 0, \tag{2}$$

$$z(t) = \phi(t) \quad t \in [-\mu, 0], \tag{3}$$

where

$x(t) \in \mathbb{R}^n$ is the state variable,

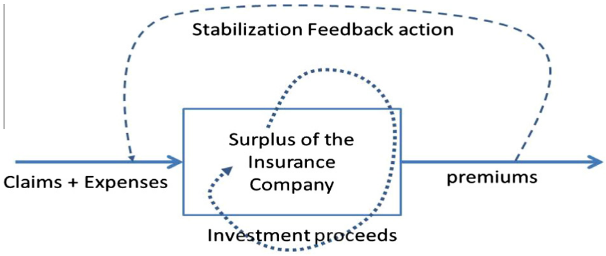

Fig. 1. Control system design of an insurance system.