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A two-warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging

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ABSTRACT

This paper deals with an inventory model for single deteriorating item with two separate warehouses (one is OW and other is RW) having different preserving facilities. Demand is assumed to be known and constant. Shortages are allowed and partially backlogged with a rate dependent on the duration of waiting time up to the arrival of next lot. Here, we have considered the permissible delay in payments. However, the model formulation is different rather than that of existing models. Accordingly, several realistic cases, sub cases and scenarios have been taken into account and the corresponding problems have been formulated as non-linear constrained optimization problems along with the solution procedure. Further, to illustrate the model and also to test the validity of the same, two numerical examples have been solved. Finally, considering first example, sensitivity analyses have been performed to study the effects of changes of different parameters, like demand, ordering cost and own warehouse capacity on optimal policies.

1. Introduction

Due to globalization of market economy, there is a fierce competition among the wholesalers/suppliers to promote their business. As a result, they offer different types of facilities to their retailers. Among these facilities, one such facility is to offer to sell a large volume of goods on credit. In that case, basically they offer a certain credit period. During this period, no interest is charged by the supplier. However, beyond this period, a higher rate of interest is charged by the supplier under certain terms and conditions by an agreement between retailer and supplier. This type of inventory problem is known as inventory problem with permissible delay in payments. According to the existing literature, this type of problem was first discussed by Haley and Higgins [1]. Then Goyal [2] developed an economic order quantity (EOQ) model under the conditions of permissible delay in payments. However, in real life situations, there are some commonly used physical goods like wheat, paddy or any other type of food grains, vegetables, fruits, drugs, pharmaceuticals etc., in which a certain fraction of these goods are damaged or decayed or vaporized or affected by some other factors and these are not in a condition to satisfy the demand. As a result, the loss due to this natural phenomenon can not be ignored in the analysis of inventory system. Aggarwal and Jaggi [3] extended the Goyal's model (EOQ model under permissible delay in payments) for deteriorating items. Jamal et al. [4] further generalized the model by allowing the completely backlogged shortages. Thereafter, a lot of works have been done by several researches. In this connection, the works of Hwang and Shinn [5], Chang et al. [6], Abad and Jaggi [7],

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Ouyang et al. [8], Huang [9–11], Liao [12], Sana and Chaudhari [13], Huang and Hsu [14], Ho et al. [15], Jaggi and Khanna [16], Jaggi and Kausar [17], Jaggi and Mittal [18] and others are worth-mentioning. However, in their works, they have developed the model for a single warehouse under the assumption that the available warehouse has unlimited capacity. This assumption is not realistic as a warehouse is of limited capacity. So, when an enterprise or a retailer purchases a large quantity of goods at a time. These goods cannot be stored in the existing warehouse (viz. OW) with limited capacity. Then for storing the excess goods, one (sometimes more than one) more warehouse (viz. RW), located away from or nearer to OW, is hired on rental basis. Usually the holding cost in RW is higher than that in OW due to the availability of better preserving facility which results a lower deterioration for the goods than OW. Hence in order to reduce the holding cost, it is more economical to consume the goods of RW at the earliest. Consequently, the management stores goods in OW first before RW, but clears the stocks in RW before OW by transferring the same from RW to OW in either continuous or bulk release pattern. This system is known as two warehouse inventory system. During last few decades, a number of research papers in this interesting area have been published by several researchers. In this connection, one may refer to the recent works of Das et al. [19], Niu and Xie [20], Rong et al. [21], Dey et al. [22], Hsieh et al. [23], Maiti [24], Lee and Hsu [25], Jaggi and Verma [26,27], Jaggi et al. [28], Jaggi et al. [29], Bhunia et al. [30] and others.

Recently, considering permissible delay in payments, Chung and Huang [31] and Liang and Zhou [32] developed two warehouse inventory models for deteriorating items. In both the works, demand is considered as constant and shortages are not allowed. In business world, stock-out situation plays an important role. Due to some unavailable circumstances, stock-out situation may occur in any business. According to the literature of inventory control theory, most of the inventory models were developed under the assumption "shortages are allowed and completely backlogged". In practice, this assumption is not realistic. Generally, customers are not interested to wait for a long time to purchase goods from a particular shop. Only a fraction of the customers will wait to purchase the good from a particular shop due to good behavior of the retailer, genuine price, quality of the goods and also the locality of the shop. As a result, shortages are considered as partially backlogged with a rate dependent on the length of waiting time up to the arrival of fresh lot.

In this paper, a two warehouse inventory model with partially backlogged shortages has been developed for single deteriorating item considering permissible delay in payments. However, the model formulation is different rather than that of existing models. In this formulation, several cases, sub cases and situations have been considered. Accordingly, the corresponding constrained optimization problems have been formulated and discussed the solution procedure. Then, to illustrate the model and its validity, two numerical examples have been considered and solved. Finally, to the study the effect of changes of ordering cost, demand and own warehouse capacity on the optimal policies of the model, sensitivity analyses have been performed considering first example.

2. Assumptions and notations

The following assumptions and notations have been used in the entire paper.

Assumptions:

(i) Replenishment rate is infinite and lead time is constant.
(ii) The inventory planning horizon is infinite and the inventory system involves only one item.
(iii) The entire lot size is delivered in one batch.
(iv) The goods of RW are transported from RW to OW in continuous release pattern. The time lag between selling from OW and filling up the space by new units from RW is negligible.
(v) The demand rate is known and constant.
(vi) Deterioration is considered only after the inventory stored in the warehouse. There is neither repair nor replacement of the deteriorated units during the inventory cycle.
(vii) Shortages, if any, are allowed and partially backlogged. During the stock-out period, the backlogging rate is dependent on the length of the waiting time up to the arrival of fresh lot. Considering this situation, the rate is defined as $[1+\delta(T-t)]^{-1}$, $\delta > 0$.

Notations:

$Q_r(t)$, $Q_o(t)$	instantaneous inventory levels at time t in RW and OW, respectively
S	highest stock level at the beginning of the cycle
R	highest shortage level
W	storage capacity of OW
$\alpha, \beta (\alpha > \beta)$	deterioration rates in OW and RW, respectively and $0 < \alpha, \beta << 1$
c_t	transportation cost per unit for transferring the items from RW to OW
A	replenishment cost (ordering cost) for replenishing the items
δ	backlogging parameter