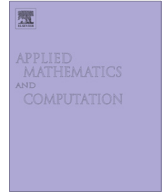




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Quadratic model of inter-population interaction: Investigation of stability areas

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ABSTRACT

A development quadratic model of the heterogeneous biological population consisting of a few sub-populations is investigated. The classic stabilization problem of solutions for the system of ordinary quadratic differential equations, which describing this model, is solved. Examples are given.

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1. Introduction

The logistic equation

$$\dot{x}(t) = ax(t) \left(1 - \frac{x(t)}{k} \right), \quad x(t) \geq 0, \quad a > 0, \quad k > 0 \quad (1)$$

is a very classical (but still actual) tool for description of population growth [1]. The equation is based on the use of two fundamental population characteristics: a is the coefficient of productivity, which characterizes potential intensity of population's growth, and k is the "environment volume". This coefficient reflects an equilibrium size of the population, which can be realized in considering environment. Naturally, the description of a population by two positive real numbers is an essential simplification. Nevertheless this simplification is quite useful and popular [2,3].

One of factors, which are not taken into account in the model (1), is heterogeneity of populations. It is clear that different specimens have different abilities, both of growth rate and potential density. In (1) one considers average values; it is quite correct if there is some normal distribution of the indices. The real situation is usually more complex: a population can be divided for some more or less discrete groups (sub-populations) and indices within each of them are more similar each other than between the groups. Forming the population these groups are not completely isolated; it is very important that they can "exchange" by specimens (both directly and through reproduction).

Each i th subpopulation is still characterized by one value k_i ($i = 1, \dots, n$; n is a number of the groups) which means stable size of the subpopulation in the environment (in the case of absence of other sub-populations!). In the same time the growth coefficient should reflect possibility of production by this subpopulation of new members for other sub-populations. One should consider a number of a_{ij} ($i, j = 1, \dots, n$), which reflects an intensity of production of specimens of j th subpopulation by i th one ($i \neq j$). The coefficient a_{ii} ; $i = 1, \dots, n$, describes the growth ("self-support") of i th subpopulation. It is quite logical to propose for the description of general dynamics of the heterogeneous biological population the following system equations:

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