Applied Mathematics and Computation 230 (2014) 200–207

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical solution of various cases of Cauchy type singular integral equation

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ARTICLE INFO

Keywords:
Cauchy type integral equation
Singular integral equation
Bernstein polynomials

ABSTRACT

In this paper, a robust but very simple numerical method is developed to solve various cases of Cauchy type singular integral equation. For this, first Bernstein polynomials are defined which are used for approximation of solution of the given singular integral equation. Then numerical method is introduced by using Bernstein polynomials. This ultimately leads to solution of a system of linear algebraic equations. Examples are illustrated to demonstrate simplicity of proposed method. Results are also compared with those present in literature to claim better efficiency of the method introduced.

1. Introduction

The Cauchy type singular integral equation is having wide application in many branches of science and engineering like aerodynamics [1] and fracture mechanics [2] etc. The Cauchy type singular integral equation

$$\int_{-1}^{1} \frac{\psi(t)}{t-x} dt + \int_{-1}^{1} K(x,t)\psi(t)dt = h(x), \quad -1 < x < 1, \tag{1}$$

is defined as generalized airfoil equation in [3] because when $K(x,t) = 0$ in Eq. (1), it is reduced to the following airfoil equation in aerodynamics

$$\int_{-1}^{1} \frac{\psi(t)}{t-x} dt = h(x), \quad -1 < x < 1. \tag{2}$$

The complete analytical solution of (2) is described in [4] for four different cases and depending upon the case taken, let it be denoted by

$$\psi(x) = \psi_k(x), \tag{3}$$

where $k = 1, 2, 3, 4$ represents the *Case I*, *Case II*, *Case III*, *Case IV*, respectively.

Case I. The solution is unbounded at both the end points $x = \pm 1$

$$\psi_1(x) = -\frac{1}{\pi^2\sqrt{1-x^2}} \int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-x} h(t)dt + \frac{C}{\sqrt{1-x^2}}, \tag{4}$$

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http://dx.doi.org/10.1016/j.amc.2013.12.114

where

$$\int_{-1}^{1} \psi_1(t)dt = C. \tag{5}$$

Case II. The solution is bounded at both the end points $x = \pm 1$

$$\psi_2(x) = -\frac{\sqrt{1-x^2}}{\pi^2} \int_{-1}^{1} \frac{h(t)}{(t-x)\sqrt{1-t^2}} dt, \tag{6}$$

provided that

$$\int_{-1}^{1} \frac{h(t)}{\sqrt{1-t^2}} dt = 0. \tag{7}$$

Case III. The solution is bounded at the end point $x = -1$, but unbounded at $x = 1$

$$\psi_3(x) = -\frac{1}{\pi^2} \sqrt{\frac{1+x}{1-x}} \int_{-1}^{1} \sqrt{\frac{1-t}{1+t}} \frac{h(t)}{(t-x)} dt. \tag{8}$$

Case IV. The solution is bounded at the end point $x = 1$, but unbounded at $x = -1$

$$\psi_4(x) = -\frac{1}{\pi^2} \sqrt{\frac{1-x}{1+x}} \int_{-1}^{1} \sqrt{\frac{1+t}{1-t}} \frac{h(t)}{(t-x)} dt. \tag{9}$$

A variety of good numerical techniques for solving Cauchy singular integral equations are available in literature but search for better is always there. For the same purpose, Kim [5] has calculated numerical solutions of Cauchy singular integral equations based on quadrature and collocation method. He has shown that the coefficient matrix of the overdetermined system obtained by taking more collocation points than quadrature nodes have the generalized inverses. Mohankumar and Natarajan [6] have shown solution prescription that combines a polynomial expansion for the unknown, a collocation procedure for fixing the expansion coefficients and a double exponential quadrature for the Cauchy principal value integral. A method based on polynomial approximation using Bernstien polynomial basis to obtain approximate numerical solution of a singular integro-differential equations with Cauchy kernel is proposed by Bhattacharya and Mandal [7]. Many researchers [8–12] recently used Bernstien polynomial basis due to its wide applicability in solving different type of differential and integral equations. Bonis and Laurita [13] have proposed a Nyström method to approximate the solutions of Cauchy singular integral equations with constant coefficients having a negative index. Further, Eshkuvatov, Nik-Long and Abdulkawi [14] have introduced an approximate solution of Cauchy type singular integral equation of first kind over $[-1, 1]$. Depending upon the weight function corresponding to the case taken, they have used Chebyshev polynomial of first kind, second kind, third kind and fourth kind. Recently, the numerical solution of a class of systems of Cauchy singular integral equations with constant coefficients is dealt by Bonis and Laurita [15]. Most recently Liu, Zhang and Wu [16] investigated the composite midpoint rule for the evaluation of Cauchy principal value integral in an interval and place the key point on its pointwise superconvergence phenomenon whereas Panja and Mandal [17] devised a method based on the Daubechies scale function to solve numerically a class of second kind integral equations with a Cauchy type kernel.

In this paper, a numerical method for approximate solution of Cauchy type singular integral equation of first kind over $[-1, 1]$ is proposed and it is based on Bernstein polynomial.

2. Definition of Bernstein polynomial basis

The Bernstein polynomials of degree n on the interval $[-1, 1]$ are defined as

$$B_{i,n}(x) = \binom{n}{i} \frac{(1+x)^i (1-x)^{n-i}}{2^n}, \quad \text{for} \quad i = 0, 1, 2, \cdots, n. \tag{10}$$

These are $(n+1)$ number of nth degree Bernstein polynomials which form a basis for linear vector space X_n, consisting of all polynomials of degree $\leqslant n$ on the interval $[-1, 1]$.

3. Method of numerical solution

Based on the known form of analytical solution, the solution can be assumed as follows:

$$\psi_k(x) = w_k(x)\chi(x), \quad k = 1, 2, 3, 4, \tag{11}$$

where $\chi(x)$is a well-behaved function on the interval $-1 \leqslant x \leqslant 1$ and