Applied Mathematics and Computation 230 (2014) 248–258

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Efficient and non-reflecting far-field boundary conditions for incompressible flow calculations

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ARTICLE INFO

*Keywords:*
Artificial compressibility
Navier–Stokes equations
Far-field boundary conditions
Characteristics method

ABSTRACT

Traditionally, the artificial compressibility (AC) method of Chorin is used for simulation of low-speed and incompressible flows. In this method, the difficulty of continuity and momentum equations decoupling is removed by adding an artificial time derivative of pressure to the continuity equation. For the first time, a fully two-dimensional upwind scheme was presented for AC equations by using characteristic structure of equations by the authors. In this paper, a new remote boundary calculation method is presented by using the idea of characteristics for these equations. Instead of simple boundary conditions which usually are employed for incompressible flows, the flow quantities at the far-field boundaries are evaluated by compatibility relations of characteristic equations. This is implemented by assuming a row of ghost cells outside of the far-field boundary and using the flux calculation method based on characteristics similar to the inside of computational domain. The idea in conjunction with multidimensional characteristic based scheme was tested for incompressible flow around circular cylinder in comparison with conventional far-field boundary condition and showed good improvements in the terms of accuracy and convergence speed.



## 1. Introduction

The method of solving low-speed or incompressible flows by the artificial compressibility (AC) correction was first introduced by Chorin [1] in obtaining steady state solutions. In this method, a time derivative of the pressure is added to the continuity equation and a coupling system of equations for pressure and velocity is obtained. Due to coupled nature of AC equations (like compressible flow equations), it is possible that the incompressible flow equations can be solved by similar methods which are used in the case of compressible flows. By reviewing the literature in this case, it is found that different schemes for discretization of AC equations have been used. They include central schemes [2], Godunov-type schemes [3–6] or characteristic based schemes [7–12].

By using the multidimensional characteristic structure of AC equations and their compatibility relations, the first multi-dimensional characteristic based scheme (MCB) for incompressible flows was introduced by the author [13,14]. Because of using multidimensional characteristic paths which the information is propagated along them, the MCB scheme takes into account the real two-dimensional nature of flow and presents remarkable superiority in the case of numerical accuracy and convergence speed [13,14].

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http://dx.doi.org/10.1016/j.amc.2013.12.089

The problem of treating open or far-field boundaries in the case of incompressible flows is still a challenging one. Many efforts have been devoted to the design of "absorbing" or "non-reflecting" boundary conditions to be applied at the outer boundary. Jin and Braza [15] proposed a nonreflecting outlet boundary condition based on a wave equation for incompressible viscous flows. The method was tested for an elliptic unsteady free shear flow and showed improvements in comparison with the other boundary methods. By using the approximate boundary layer equations for wall-bounded flows, Fournier et al. [16] proposed a new outflow boundary condition for these flows that was demonstrated to reproduce the exact Blasius solution over the numerical domain. Sani and Saidi [17] used a lagged implicit data reconstruction procedure in combination with overall mass conservation enforcement for introducing a new way for open boundaries estimation.

Recently, fast convergence and less computational effort are obtained by implementing of Graphics Processing Unit (GPU) for numerical computations. Itu et al. used an optimized GPU based simulation for the solution of incompressible flow over a backward facing step inside a channel [18].

In the case of AC method for solving the incompressible flows, the velocity components and pressure at the far-field and open boundaries are estimated usually by specifying from the free stream or interpolating from the domain. This is defined with respect to the characteristic waves traveling in and out of the numerical domain. For example, at the far-field outflow boundary, assuming that the fluid leaving the domain is traveling in the positive direction, there are two characteristic waves traveling out of the computational domain, i.e., $\lambda_0 > 0,\ \lambda_1 > 0$ and $\lambda_2 < 0$, in which $\lambda_0,\ \lambda_1,\ \lambda_2$ are the eigenvalues of AC system of equations. Therefore the velocity components $(u,v)$ are interpolated from the interior cells of domain and the pressure is fixed by the value of free stream pressure. For more details see [19]. This method of defining boundary conditions at the far-field boundaries has been used by many researchers in the case of incompressible flow computations by artificial compressibility method [7,13,14,20,21].

In the present paper, by using the approach of multidimensional characteristics for AC equations, a new method for estimating the flow variables at the far-field boundary is presented which is in a consistent manner with the MCB scheme. By the numerical experiments, it was found that the conventional far-field boundary estimation for incompressible flows will cause reflected waves to be returned back to the computational domain and delay the solution to steady state whereas the proposed method takes into account the real multidimensional nature of flow near the boundaries which provides non-reflecting outer boundaries and fast convergence.

## 2. Governing equations

The Navier–Stokes equations for two-dimensional incompressible flows modified by artificial compressibility correction can be expressed as

$$\int\int_\Omega \frac{\partial \mathbf{W}}{\partial t}\,dV + \oint_C (\mathbf{F}\,dS_x + \mathbf{G}\,dS_y) = \frac{1}{\mathrm{Re}}\oint_C (\mathbf{R}\,dS_x - \mathbf{S}\,dS_y) \tag{1}$$

where

$$\mathbf{W} = \begin{bmatrix} p \\ u \\ v \end{bmatrix},\quad \mathbf{F} = \begin{bmatrix} \beta u \\ u^2 + p \\ uv \end{bmatrix},\quad \mathbf{G} = \begin{bmatrix} \beta v \\ uv \\ v^2 + p \end{bmatrix},$$
$$\mathbf{R} = \begin{bmatrix} 0 \\ \partial u/\partial x \\ \partial v/\partial x \end{bmatrix},\quad \mathbf{S} = \begin{bmatrix} 0 \\ \partial u/\partial y \\ \partial v/\partial y \end{bmatrix}.$$

Here $\mathbf{W}$ is the vector of primitive variables, and $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{R}$, $\mathbf{S}$ are convective and viscous flux vectors, respectively. The artificial compressibility parameter and Reynolds number are shown as $\beta$ and Re, respectively. The discretized form of Eq. (1) reads:

$$A_{ij}\frac{\partial \mathbf{W}_{ij}}{\partial t} + \sum_{k=1}^{4}(\mathbf{F}\,\Delta S_x + \mathbf{G}\,\Delta S_y)_k = \frac{1}{\mathrm{Re}}\sum_{k=1}^{4}(\mathbf{R}\,\Delta S_x - \mathbf{S}\,\Delta S_y)_k \tag{2}$$

where $A_{ij}$ is the cell area.

## 3. Characteristic paths and compatibility relations for two-dimensional AC equations

Characteristic and compatibility relations for AC equations are derived by considering their corresponding "Euler equations" as follows [20]:

$$\begin{cases} p_t + \beta u_x + \beta v_y = 0 \\ u_t + uu_x + vu_y + p_x = 0 \\ v_t + uv_x + vv_y + p_y = 0 \end{cases} \tag{3}$$