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An improved family of estimators of finite population mean based on the auxiliary attribute

Abdul Haq *, Javid Shabbir

Department of Statistics, Quaid-i-Azam University, Islamabad 45320, Pakistan

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ABSTRACT

Recently, Koyuncu (2012) [3] proposed an efficient family of estimators for estimation of finite population mean using information on the auxiliary attribute. In this paper, we propose two improved estimators of finite population mean. The biases and mean squared errors of the proposed estimators are derived up to the first order of approximation. It is observed that the first proposed estimator is always better than the first family of estimators adapted by Koyuncu (2012) [3]. An empirical study is carried out to demonstrate the performance of the proposed estimators.

1. Introduction

The auxiliary information is frequently used to increase precision of the estimates by taking advantage of correlation between the study variable and the auxiliary variable. Another way to increase the efficiency of the ratio estimator is to use the information on the auxiliary attributes. Several authors, including Koyuncu [3], Shabbir and Gupta [6], [7] Naik and Gupta [5], Abd-Elfattah et al. [1], have proposed improved estimators of finite population mean using information on an auxiliary attribute.

Consider $\Omega = \{\Omega_1, \Omega_2, \ldots, \Omega_i \ldots, \Omega_N\}$ be a finite population of size N. A sample of size n is selected from Ω by using simple random sampling without replacement. Let y_i and φ_i denote the values of the study variable and the binary auxiliary attribute for the ith unit of the population, respectively. Here it is assumed that φ_i can take only two possible values, depending on the presence of an attribute, say φ, i.e.,

$$\varphi_i = 1, \quad \text{if the } i\text{th unit of the population possesses attribute } \varphi,$$
$$\varphi_i = 0, \quad \text{otherwise}.$$

Let $\Pi = \sum_{i=1}^{N} \varphi_i$ and $\pi = \sum_{i=1}^{n} \varphi_i$ denote the total number of units in the population and in the sample, respectively, possessing an auxiliary attribute φ. The corresponding population and sample proportions are $P = \frac{\Pi}{N}$ and $p = \frac{\pi}{n}$, respectively. Similarly, let $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ be the population and the sample means of the study variable y, respectively. In order to estimate the population mean $\overline{Y}$, we assume that P is known. Let $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ and $s_\varphi^2 = \frac{np(1-p)}{n-1}$ be the sample variances corresponding to the population variances $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \overline{Y})^2$ and $S_\varphi^2 = \frac{NP(1-P)}{N-1}$, respectively. Let ρ_{pb} be the correlation coefficient between the study variable y and the auxiliary attribute φ. Let $C_y = \frac{S_y}{\overline{Y}}$ and $C_\varphi = \frac{S_\varphi}{P}$ be the coefficients of variation of y and φ, respectively. In order to find the biases and mean squared errors (MSEs) of the estimators, we define the following relative error terms.

* Corresponding author.
E-mail address: aaabdulhaq@yahoo.com (A. Haq).

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Let $\xi_0 = \frac{\bar{y}}{\bar{Y}} - 1$ and $\xi_1 = \frac{p}{P} - 1$, such that E(ξ_i) = 0, for i = 0, 1, where E(·) represents the mathematical expectation. Let $E(\xi_0^2) = \theta C_y^2 = V_{20}, E(\xi_1^2) = \theta C_\varphi^2 = V_{02}$ and $E(\xi_0\xi_1) = \theta\rho_{pb}C_yC_\varphi = V_{11}$, where $V_{rs} = \frac{E\{(\bar{y}-\bar{Y})^r(p-P)^s\}}{\bar{Y}^rP^s}$, for $r, s = 0, 1, 2, \theta = \frac{1-f}{n}, f = \frac{n}{N}$.

In this paper, we have proposed two improved estimators of finite population mean by modifying the estimators suggested by Koyuncu [3]. The biases and MSEs of the proposed estimators are derived up to the first order of approximation. Two real data sets are used for numerical comparisons. It is worth mentioning that the proposed estimators are more efficient than the estimators suggested by Koyuncu [3].

The rest of the paper is as follows: Section 2 includes several estimators of finite population mean based on an auxiliary attribute. In Section 3, we find the biases and MSEs of the proposed estimators. Section 4 contains numerical comparisons of the proposed and the existing estimators. Section 5 finally summarizes the main findings.

2. Estimators available in literature

In this section, we review the estimators of finite population mean.

2.1. Unbiased estimator

The traditional unbiased estimator of finite population mean is

$$\hat{\bar{Y}}_U = \bar{y}. \tag{1}$$

The variance of $\hat{\bar{Y}}_U$ is given by

$$\mathrm{Var}(\hat{\bar{Y}}_U) = \bar{Y}^2V_{20}. \tag{2}$$

2.2. Difference estimator

The difference estimator of finite population mean based on an auxiliary attribute is

$$\hat{\bar{Y}}_D = \bar{y} + k(P - p), \tag{3}$$

where k is an unknown constant, and its value is determined such that it minimizes the variance of $\hat{\bar{Y}}_D$.

The variance of $\hat{\bar{Y}}_D$ is given by

$$\mathrm{Var}(\hat{\bar{Y}}_D) = \bar{Y}^2V_{20} + k^2P^2V_{02} - 2kYPV_{11}. \tag{4}$$

The minimum variance of $\hat{\bar{Y}}_D$, at optimum value of k, i.e., $k_{(\mathrm{opt})} = \frac{\bar{Y}V_{11}}{PV_{02}}$, is given by

$$\mathrm{Var}_{\min}(\hat{\bar{Y}}_D) = \bar{Y}^2V_{20}(1 - \rho_{pb}^2), \tag{5}$$

where $\rho_{pb} = \frac{V_{11}}{\sqrt{V_{20}V_{02}}}$ is the point bi-serial correlation coefficient.

2.3. Koyuncu [3] adapted family of estimators

Gupta and Shabbir [2] proposed a family of estimators for the population mean by using information on an auxiliary variable in simple random sampling. Koyuncu and Kadilar [4] considered the same family of estimators under stratified random sampling. On similar lines, Koyuncu [3] proposed a family of estimators by using information on an auxiliary attribute in simple random sampling, given by

$$\hat{\bar{Y}}_N = \{t_1\bar{y} + t_2(P - p)\}\left(\frac{\eta P + \lambda}{\eta p + \lambda}\right), \tag{6}$$

where t_1 and t_2 are two unknown constants, whose values are to be determined such that the MSE of $\hat{\bar{Y}}_N$ is minimum. Here η and λ are either real numbers or functions of the known parameters of the auxiliary attribute. Some members of $\hat{\bar{Y}}_N$ are given in Table 1.

Table 1
Some members of the existing and proposed family of estimators.

$\hat{\bar{Y}}_N$	$\hat{\bar{Y}}_{P1}$	η	λ
$\hat{\bar{Y}}_{N(1)}$	$\hat{\bar{Y}}_{P1(1)}$	C_p	$\beta_{2(\varphi)}$
$\hat{\bar{Y}}_{N(2)}$	$\hat{\bar{Y}}_{P1(2)}$	$\beta_{2(\varphi)}$	C_p
$\hat{\bar{Y}}_{N(3)}$	$\hat{\bar{Y}}_{P1(3)}$	1	C_p
$\hat{\bar{Y}}_{N(4)}$	$\hat{\bar{Y}}_{P1(4)}$	1	$\beta_{2(\varphi)}$