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Analysis of a delayed Chlamydia epidemic model with pulse vaccination

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ABSTRACT

In this paper, we have considered a dynamical model of Chlamydia disease with varying total population size, saturation incidence rate and discrete time delay to become infectious. It is assumed that there is a time lag (τ) to account for the fact that an individual infected with bacterium *Chlamydia trachomatis* is not infectious until after some time after exposure. The probability that an individual remains in the latency period (exposed class) at least t time units before becoming infectious is given by a step function with value 1 for $0 \leqslant t < \tau$ and value zero for $t > \tau$. The probability that an individual in the latency period has survived is given by $e^{-\mu\tau}$, where μ denotes the natural mortality rate in all epidemiological classes. Pulse vaccination is an effective and important strategy for the elimination of infectious diseases and so we have analyzed this model with pulse vaccination. We have defined two positive numbers R_1 and R_2. It is proved that there exists an infection-free periodic solution which is globally attractive if $R_1 < 1$ and the disease is permanent if $R_2 > 1$. Our analytical findings are illustrated through computer simulation using MATLAB, which show the reliability of our model from the epidemiological point of view.

1. Introduction

Infectious diseases have tremendous influence on human life and are usually caused by pathogenic microorganisms, such as bacteria, viruses, parasites, or fungi; the diseases can be spread directly or indirectly. Sexually transmitted infections (STIs) remain a major public health challenge globally and are among the most common infections in the United States. Chlamydia, caused by the bacterium *Chlamydia trachomatis*, is one of the most important sexually-transmitted infections worldwide. It is a major and the commonest sexually-transmitted bacterial disease in European countries [19,32] and the United States [36,41]. It was estimated that around 92 million Chlamydia infections occurred worldwide in 1999, affecting more women (50 million) than men (42 million) [5,51]. In 2007, 1,108,374 Chlamydia diagnoses were reported in the United States, up from 1,030,911 in 2006. The 2007 total represents the largest number of cases ever reported to the Centers for Disease Control and Prevention (CDC) for any condition [37].

C. trachomatis is sexually-transmitted (via vaginal, oral and anal sex) and it is also transmitted through other modes, such as vertically (from mother to child) [12]. This disease causes numerous irreversible complications such as chronic pelvic pain, infertility in females and pelvic inflammatory disease. Chlamydia can be easily treated and cured, if diagnosed effectively, with antibiotics. But usually occurs without symptoms and often goes undiagnosed. Untreated, it can cause severe health consequences for women and up to 40% of females with untreated Chlamydia infections develop pelvic inflammatory disease

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(PID), a condition which can lead to such long-term complications as infertility, ectopic pregnancy and chronic pelvic pain [26]. In pregnant women, it may lead to premature delivery and babies born to infected mothers can get infections in their eyes, called conjunctivitis or pinkeye, as well as pneumonia. Complications from Chlamydia among men are relatively uncommon, but may include epididymitis and urethritis which can cause pain, fever and in rare cases, sterility. Men with Chlamydia symptoms might have a discharge from the penis and a burning sensation when urinating which may range from clear to pussy. Men might also have burning and itching around the opening of the penis or pain and swelling in the testicles/scrotum, or both which can be a sign of epididymitis, an inflammation of a part of the male reproductive system located in the testicles. Both PID and epididymitis can result in infertility. In addition, investigations indicate that the presence of Chlamydia infection increases the risk of HIV transmission [20,28]. Chlamydia is also known as a "silent" disease since 75% of infected women and at least 50% of infected men have no symptoms. If symptoms do occur, they usually appear after 1 to 3 weeks of exposure [1,12,42]. Women, especially young and minority women, are hit hardest by *C. trachomatis* and it is found that women are most severely impacted by the long-term consequences of untreated Chlamydia. The reported Chlamydia case rate for females in 2007 was almost three times higher than for males (543.6 and 190.0 per 100,000 population respectively). Young females 15 to 19 years of age had the highest Chlamydia rate followed by females 20 to 24 years of age [37]. Some women may still have no signs or symptoms, which is the cause that it is often transmitted from one sexual partner to another without either knowing [1]. The annual cost of treatment of Chlamydia and its consequences in the United States is more than $2 billion [1,13]. *C. trachomatis* causes more cases of sexually-transmitted diseases (STDs) than any other bacterial pathogen and so making it a major public health problem throughout the world [52]. Another important fact about Chlamydia is that infected individuals can acquire re-infection while recovering from the disease [12] and often arises in situations where infected individuals have multiple sex partners. Although Chlamydia-related mortality is negligible in comparison to other STDs such as HIV/AIDS, the aforementioned Chlamydia-associated irreversible complications makes Chlamydia a disease of major public health significance [12,52,53].

The pulse vaccination strategy (PVS) consists of repeated application of vaccine at discrete time with equal interval in a population in contrast to the traditional constant vaccination [21,54]. Compared to the proportional vaccination models, the study of pulse vaccination models is in its infancy [54]. At each vaccination time a constant fraction of the susceptible population is vaccinated successfully. Since 1993, attempts have been made to develop mathematical theory to control infectious diseases using pulse vaccination [2,21]. Nokes and Swinton [39] discussed the control of childhood viral infections by pulse vaccination strategy. Stone et al. [44] presented a theoretical examination of the pulse vaccination strategy in the *SIR* epidemic model and d'Onofrio [16,17] analyzed the use of pulse vaccination policy to eradicate infectious disease for *SIR* and *SEIR* epidemic models. Different types of vaccination policies and strategies combining pulse vaccination policy, treatment, pre-outbreak vaccination or isolation have already been introduced by several researchers [6,18,22–24,47,50].

Mathematical epidemiology is the study of the spread of diseases, in space and time, with the objective to identify factors that are responsible for or contributing to their occurrence. Mathematical models are becoming important tools in analyzing the spread and control of infectious diseases. Epidemic models of ordinary differential equations have been studied by a number of researchers [4,9–11,15,29,31,34,35,38,48]. The basic and important objectives for these models are the existence of the threshold values which distinguish whether the infectious disease will be going to extinct, the local and global stability of the disease-free equilibrium and the endemic equilibrium, the existence of periodic solutions and the persistence of the disease. Stability, persistence and permanence in population biology have been studied by many researchers [45,46]. Hence, as a part of population biology, permanence of disease plays an important role in mathematical epidemiology.

Although Chlamydia is a disease of significant public health importance, not much has been analyzed in terms of using mathematical modeling to gain insight into its transmission dynamics at population level [42]. In this paper we have used the Kermack–McKendrick compartmental modeling framework, which entails sub-dividing the entire high-risk human population into mutually-exclusive epidemiological compartments (based on disease status), to gain insights into the qualitative features of *C. trachomatis* in a human population (with the aim of finding effective ways to control its spread). The main feature of this paper is to introduce time delay, saturation incidence rate with valid pulse vaccination strategy. We have introduced two threshold values R_1 and R_2 and further obtained that the disease will be going to extinct when $R_1 < 1$ and the disease will be permanent when $R_2 > 1$. All our important findings are numerically verified using MATLAB in Section 5. The aim of the analysis of this model is to trace the parameters of interest for further study, with a view to informing and assisting policy-maker in targeting prevention and treatment resources for maximum effectiveness.

2. Model derivation and preliminaries

In the following, we consider a dynamical model of Chlamydia disease that spread by *C. trachomatis* (a type of bacteria) with discrete time delay and pulse vaccination strategy (PVS) which satisfies the following assumptions:

The underlying high-risk human population is split up into five mutually-exclusive classes (compartments), namely, susceptible (S), exposed (infected but not yet infectious) (E), infective in asymptomatic phase (showing no symptoms of Chlamydia) (I_A), infective in symptomatic phase (showing symptoms of Chlamydia) (I_S) and recovered (infectious people who have cleared (or recovered from) Chlamydia infection) (R).

The susceptible population increases by the recruitment through new sexually-active individuals, migration and recovered hosts and decreases due to direct contact with infected individuals, natural death and pulse vaccination strategy.