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Necessary and sufficient conditions for permanence and extinction in a three dimensional competitive Lotka–Volterra system ☆

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ABSTRACT

A three dimensional nonautonomous competitive Lotka–Volterra system is considered in this paper. It is shown that if the growth rates are positive, bounded and continuous functions, and the averages of the growth rates satisfy certain inequalities, then any positive solution has the property that one of its components vanishes. Moreover, if one of the above inequalities is changed, then all components of any positive solution have positive infimum.

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1. Introduction

In [1], Ahmad and Lazer considered the nonautonomous competition Lotka–Volterra system

$$u_k' = \frac{du_k}{dt} = u_k(t)\left[a_k(t) - \sum_{j=1}^{N} b_{kj}u_j(t)\right], \quad k = 1, \dots, N, \tag{1.1}$$

where the growth rates $a_k(t)$ are continuous and positive on R and b_{kj} are nonnegative constants for $1 \leqslant j, k \leqslant N$ with $b_{kk} > 0$ for $1 \leqslant k \leqslant N, R = (-\infty, +\infty)$. Let a_{kM} and a_{kL} denote the supremum and the infimum of $a_k(t)$ respectively for $t \in R$ and $1 \leqslant k \leqslant N$. R^N denotes the N-dimensional real Euclidean space, and R_+^N denotes the nonnegative cone of R^N. Assume that the growth rates have the property that

$$\lim_{T\to+\infty} \frac{1}{T}\int_{t_0}^{t_0+T} a_k(t)dt = M[a_k]$$

exists for $1 \leqslant k \leqslant N$ and these limits are uniform with respect to $t_0 \in R$, they showed that the conditions

$$a_{kL} > \sum_{j=1, j\neq k}^{N} b_{kj}a_{jM}/b_{jj}, \quad 1 \leqslant k \leqslant N-1 \tag{1.2}$$

imply that system of linear equations

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$$M[a_k] = \sum_{j=1}^{N-1} b_{kj} x_j, \quad 1 \leqslant k \leqslant N-1$$

has a unique solution $x_1 = \xi_1^*, x_2 = \xi_2^*, \ldots, x_{N-1} = \xi_{N-1}^*$ and the numbers $\xi_1^*, \ldots, \xi_{N-1}^*$ are positive. They also showed that if conditions (1.2) and

$$M[a_N] < \sum_{j=1}^{N-1} b_{Nj} \xi_j^*, \tag{1.3}$$

hold, then $\lim_{t\to+\infty} u_N(t) = 0$ for any solution $u(t) = \mathrm{col}(u_1(t), \cdots, u_N(t))$ of (1.1) with $u_k(t_0) > 0$ for $1 \leqslant k \leqslant N$. On the other hand if conditions (1.2) hold, then

$$M[a_N] > \sum_{j=1}^{N-1} b_{Nj} \xi_j^* \tag{1.4}$$

is both a necessary and sufficient condition that $\inf_{t \geqslant t_0} u_k(t) > 0, 1 \leqslant k \leqslant N$, for any solution $u(t) = \mathrm{col}(u_1(t), \ldots, u_N(t))$ of (1.1) with $u_k(t_0) > 0$ for $1 \leqslant k \leqslant N$. They supposed that the above results remain true if conditions (1.2) are replaced by weaker inequalities

$$M[a_k] > \sum_{\substack{j=1 \\ j \neq k}}^{N} b_{kj} M[a_j]/b_{jj}, \quad 1 \leqslant k \leqslant N-1. \tag{1.5}$$

But they only gave the proof for the case $N = 2$, i.e. if

$$M[a_1] > b_{12} M[a_2]/b_{22} \tag{1.6}$$

and

$$M[a_2] < b_{21} M[a_1]/b_{11}$$

hold, then $\lim_{t\to+\infty} u_2(t) = 0$ for any solution $u(t) = \mathrm{col}(u_1(t), u_2(t))$ of (1.1) with $u_k(t_0) > 0$, $k = 1, 2$. On the other hand, if (1.6) and

$$M[a_2] > b_{21} M[a_1]/b_{11}$$

hold, then $\inf_{t \geqslant t_0} u_k(t) > 0, k = 1, 2$, for any solution $u(t) = \mathrm{col}(u_1(t), u_2(t))$ of (1.1) with $u_k(t_0) > 0, k = 1, 2$. Later, they gave the proof of the above conjecture for the case N in [2] by their earlier results established in [1,3–5].

In this paper, we give the proof of the above conjecture in [1] for the case $N = 3$ with the method of [1]. Our proof is different from that of [2]. Considering the following equations

$$\begin{cases} u_1'(t) &= u_1(t)[a_1(t) - b_{11}u_1(t) - b_{12}u_2(t) - b_{13}u_3(t)], \\ u_2'(t) &= u_2(t)[a_2(t) - b_{21}u_1(t) - b_{22}u_2(t) - b_{23}u_3(t)], \\ u_3'(t) &= u_3(t)[a_3(t) - b_{31}u_1(t) - b_{32}u_2(t) - b_{33}u_3(t)], \end{cases} \tag{1.7}$$

where the growth rates $a_1(t), a_2(t), a_3(t)$ are continuous and positive on R and b_{kj} are nonnegative constants for $1 \leqslant j, k \leqslant 3$ with $b_{kk} > 0$ for $1 \leqslant k \leqslant 3$.

Now we give our main result.

Theorem 1.1. *Assume that the growth rates $a_1(t), a_2(t), a_3(t)$ are bounded above and below by positive constants on R, and the growth rates have the property that*

$$\lim_{T\to+\infty} \frac{1}{T} \int_{t_0}^{t_0+T} a_k(t)dt = M[a_k]$$

exists for $1 \leqslant k \leqslant 3$ and these limits are uniform with respect to $t_0 \in R$. In addition, $\det B \geqslant 0$, where

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}.$$

(a) *If*

$$M[a_1] > b_{12} M[a_2]/b_{22} + b_{13} M[a_3]/b_{33}, \tag{1.8}$$

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