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Stability criteria for linear Hamiltonian systems under impulsive perturbations

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ABSTRACT

Stability criteria are given for planar linear periodic Hamiltonian systems with impulse effect by making use of a Lyapunov type inequality. A disconjugacy criterion is also established. The results improve the related ones in the literature for such systems.

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1. Introduction

Consider the Hamiltonian system

$$y' = JH(t)y, \quad t \in \mathbb{R}, \tag{1}$$

where $H(t)$ is a symmetric 2×2 matrix with entries $h_{jk}(t)$ and

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

By setting $y_1(t) = x(t), y_2(t) = u(t)$, $h_{12}(t) = h_{21}(t) = a(t), h_{22}(t) = b(t), h_{11}(t) = c(t)$, one can write the system (1) as

$$x' = a(t)x + b(t)u, \quad u' = -c(t)x - a(t)u, \quad t \in \mathbb{R}. \tag{2}$$

We remark that the second-order differential equation

$$(p(t)x')' + q(t)x = 0, \quad t \in \mathbb{R}$$

is a special case of (2) via

$$a(t) \equiv 0, \quad b(t) = \frac{1}{p(t)}, \quad c(t) = q(t),$$

where $p(t)$ and $q(t)$ are real-valued functions and $p(t) \neq 0$ for any $t \in \mathbb{R}$.

In what follows we assume that $a(t), b(t)$, and $c(t)$ satisfy the periodicity conditions

$$a(t+T) = a(t), \quad b(t+T) = b(t), \quad c(t+T) = c(t), \quad t \in \mathbb{R}.$$

The system (2) is said to be *stable* if all solutions are bounded on $\mathbb{R}$, *unstable* if all nontrivial solutions are unbounded on $\mathbb{R}$, and *conditionally stable* if there exits a nontrivial solution bounded on $\mathbb{R}$.

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The following well known stability theorem was given by Krein in [5].

Theorem 1.1. *If*

$$b(t) \geqslant 0, \quad c(t) \geqslant 0, \quad b(t)c(t) - a^2(t) \geqslant 0;$$

$$\int_0^T b(t)\,\mathrm{d}t \int_0^T c(t)\,\mathrm{d}t - \left(\int_0^T a(t)\,\mathrm{d}t\right)^2 > 0;$$

$$\int_0^T |a(t)|\,\mathrm{d}t + \left\{\int_0^T b(t)\,\mathrm{d}t \int_0^T c(t)\,\mathrm{d}t\right\}^{1/2} < 2,$$

then system (2) is stable.

A similar result was obtained by Guseinov and Kaymakçalan in [3] by making use of Floquet theorem [2,6].

Theorem 1.2. *If*

$$b(t) > 0, \quad c(t) \geqslant 0, \quad b(t)c(t) - a^2(t) \geqslant 0;$$

$$b(t)c(t) - a^2(t) \not\equiv 0;$$

$$\int_0^T |a(t)|\,\mathrm{d}t + \left\{\int_0^T b(t)\,\mathrm{d}t \int_0^T c(t)\,\mathrm{d}t\right\}^{1/2} < 2,$$

then system (2) is stable.

Recently, Guseinov and Zafer [4] established stability criteria for Hamiltonian systems under impulse effect of the form

$$\begin{aligned} &x' = a(t)x + b(t)u, \quad u' = -c(t)x - a(t)u, \quad t \neq \tau_i, \\ &x(\tau_i+) = \alpha_i x(\tau_i-), \quad u(\tau_i+) = \alpha_i u(\tau_i-) - \beta_i x(\tau_i-), \end{aligned} \tag{3}$$

where $t \in \mathbb{R}$ and $i \in \mathbb{Z}$; $\{\tau_i\}$ $(i \in \mathbb{Z})$ is a sequence of real numbers (impulse points) such that $\tau_i < \tau_{i+1}$ for all $i \in \mathbb{Z}$, and that

$$\tau_{i+r} = \tau_i + T\,(i \in \mathbb{Z}), \quad 0 < \tau_1 < \tau_2 < \cdots < \tau_r < T$$

for some positive real number T and some positive integer r; the functions $a, b, c : \mathbb{R} \setminus \{\tau_i : i \in \mathbb{Z}\} \to \mathbb{R}$ and the real sequences $\{\alpha_i\}, \{\beta_i\}$ $(i \in \mathbb{Z})$ satisfy the periodicity conditions:

$$\begin{aligned} &a(t+T) = a(t), \quad b(t+T) = b(t), \quad c(t+T) = c(t), \ t \in \mathbb{R} \setminus \{\tau_i : i \in \mathbb{Z}\}; \\ &\alpha_i \neq 0, \quad \alpha_{i+r} = \alpha_i, \quad \beta_{i+r} = \beta_i, \ i \in \mathbb{Z}; \\ &a, b, c \in \mathrm{PC}[0, T], \end{aligned}$$

where $\mathrm{PC}[0, T]$ denotes the set of functions

$$f : [0, T] \setminus \{\tau_1, \tau_2, \ldots, \tau_r\} \to \mathbb{R},$$

such that $f \in C([0, T] \setminus \{\tau_1, \tau_2, \ldots, \tau_r\})$ and the left-hand limit $f(\tau_i-)$ and the right-hand limit $f(\tau_i+)$ exist (finite) for each $i \in N_1^r := \{1, 2, \ldots, r\}$. As usual, by $\mathrm{PC}^1[0, T]$, we mean the set of functions $f \in \mathrm{PC}[0, T]$ such that $f' \in \mathrm{PC}[0, T]$.

The next two theorems we have extracted from [4] are of particular importance for our work in this paper. We note that the inequality (6) is given as a nonstrict inequality in [4], however it should be a strict one as pointed out in [8].

Theorem 1.3. *Assume that*

$$\prod_{i=1}^{r} \alpha_i^2 = 1; \tag{4}$$

$$b(t) > 0, \quad \int_0^T \left(c(t) - \frac{a^2(t)}{b(t)}\right)\mathrm{d}t + \sum_{i=1}^{r} \frac{\beta_i}{\alpha_i} > 0; \tag{5}$$

$$\int_0^T |a(t)|\,\mathrm{d}t + \left[\int_0^T b(t)\,\mathrm{d}t\right]^{\frac{1}{2}} \left[\int_0^T c^+(t)\,\mathrm{d}t + \sum_{i=1}^{r} \left(\frac{\beta_i}{\alpha_i}\right)^+\right]^{\frac{1}{2}} < 2, \tag{6}$$

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