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Tolerable delay-margin improvement for systems with input–output delays using dynamic delayed feedback controllers

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ABSTRACT

This paper presents investigations on a dynamic state feedback controller with state delays that improves tolerable delay margin for systems with input–output delays. Using an iterative pole placement technique for time-delay systems, the effect of introducing state delay in the controller dynamics is studied. It is observed that such a controller improves the tolerable delay margins compared to its static or even simple dynamic counterpart.

1. Introduction

Time-delay is inherent to many feedback control systems owing to the fact that information takes finite time to get transported. Often, delays appear in the feedback loop due to the time taken in (i) measuring outputs (ii) computing control actions and (iii) actuating the plant. Such delays in the feedback loop are, in general, destabilizing [1]. However, it is also possible that purposeful use of artificial delays in the controller may improve stability of certain systems, e.g., (i) use of an appropriate delay leads to chattering stability in a milling process [2], (ii) use of delay may yield better purchasing and stocking decisions in supply chain management [3]. Such stabilizing effect of delays is a motivation to many researchers to exploit the possibilities of using them with benefits.

This paper considers the problem of stabilizing systems with Input and Output (IO) delays as shown in Fig. 1. Time taken in measuring the output signal and thereby receiving at the controller is called as the output delay (τ_s), whereas the sending time for the control signal from the controller to the actuator is the input delay (τ_a). For such systems, if one uses a static feedback controller then the delay in the feedback loop may be represented as $\tau_{total} = \tau_a + \tau_s$ [4].

For an illustration, consider a scalar system of the form

$$\dot{x}(t) = ax(t) + u(t - \tau_a), \tag{1}$$

It is well known that using a static state feedback controller of the form $u(t) = k_s x(t - \tau_s)$, where k_s is the control gain, system (1) can be stabilized till $a(\tau_a + \tau_s) < 1$ [5]. However, if one uses an observer based controller of the form

$$\dot{\hat{x}}(t) = a\hat{x}(t) + k\hat{x}(t - \tau_a) + l\hat{x}(t - \tau_s) - lx(t - \tau_s), \tag{2}$$

where $\hat{x}(t)$ is the estimate of the state, l is the observer gain, then the scalar system (1) can be stabilized till $a\tau_a < 1$ and $a\tau_s < 1$ [5], which is an improvement over the static feedback one. However, implementing such a controller is difficult since

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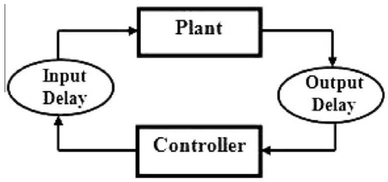

Fig. 1. Feedback control system with input–output delays.

one has to obtain accurate information of the two delays, which is impractical specifically when these delays are uncertain or time-varying.

From this perspective, it may be intuited that dynamic controller with delay might have stability improvement ability for time-delay systems. Note that the inclusion of delay in such controllers is important in addition to the dynamicness. Since, similar to systems without time delays, simple dynamic controllers without time delay doesn't have any stability improvement ability as compared to static controllers typically for state feedback case. The same has been experienced by present authors for several example cases.

From the above discussion, it may be perceived that dynamics and state delays combinedly in controllers may help in improving the tolerable delay bound. Question that now arises is whether the controller dynamics, its state delays or both of them contribute to this improvement. This paper attempts to address this question and proposes a dynamic feedback controller with state delays that improves tolerable delay bound in the feedback loop further. It is just to mention here that this work does not investigate the stabilizing ability of controller with time delays for systems that are not otherwise stabilizable, as it has been attempted in [6]. Rather, it looks into the possibility of tolerable delay margin improvement for systems that are conventionally stabilizable.

2. Problem consideration

The plant dynamics with input delay is represented as:

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t - \tau_a), \tag{3}$$

where $x_p(t) \in \Re^{n_p}$ is the state, $u_p(t) \in \Re^{m_p}$ is the control input; A_p and B_p are constant matrices of appropriate dimension. For stabilizing such system, we consider the following controller types:

Type I: Simple dynamic controller

$$\dot{x}_c(t) = A_{c0} x_c(t) + C_c x_p(t - \tau_s), \quad u_p(t) = x_c(t); \tag{4}$$

Type II: Dynamic controller with a state delay

$$\dot{x}_c(t) = A_{c0} x_c(t) + A_{c1} x_c(t - \tau_1) + C_c x_p(t - \tau_s), \quad u_p(t) = x_c(t); \tag{5}$$

Type III: Dynamic controller with two state delays

$$\begin{aligned} &\dot{x}_c(t) = A_{c0} x_c(t) + A_{c1} x_c(t - \tau_1) + A_{c2} x_c(t - \tau_2) + C_c x_p(t - \tau_s), \\ &u_p(t) = x_c(t); \end{aligned} \tag{6}$$

where $x_c(t) \in \Re^{n_c}$ is the state of the dynamic controller and $A_{c0},\ A_{c1},\ A_{c2}$ and C_c are the controller matrices to be designed.

Stabilization using Type I is of interest to study the effect of controller dynamics on improvement in tolerable delay ranges whereas the same for Type II corresponds to the effect of both the dynamics and controller state delay. Comparison of the stabilizing ability of Type III explores whether use of more than one delay in the controller states has any further effect. It may be noted that the controller of Type III is similar to the observer based controller. However, the delays τ_1 and τ_2 may take different values other than the IO delays and may be chosen appropriately.

The closed-loop system for the Type-III controller, (3) along with (6), may be written as:

$$\dot{\xi}(t) = A\xi(t) + B\xi(t - \tau_1) + C\xi(t - \tau_2) + D\xi(t - \tau_a) + E\xi(t - \tau_s), \tag{7}$$

where

$$\xi(t) = \begin{bmatrix} x_p(t) \\ x_c(t) \end{bmatrix}, \quad A = \begin{bmatrix} A_p & 0 \\ 0 & A_{c0} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & A_{c1} \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 0 \\ 0 & A_{c2} \end{bmatrix}, \quad D = \begin{bmatrix} 0 & B_p \\ 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & 0 \\ C_c & 0 \end{bmatrix}$$

Note that, the closed loop system for other types of controllers are subset of the above.