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# Extended transformed rational function method and applications to complexiton solutions

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ABSTRACT

The transformed rational function method provides a systematical and convenient handling of the solution process of nonlinear equations, unifying the tanh function type methods, the homogeneous balance method, the exp-function method, the mapping method, and the F-expansion type methods. In this paper, the transformed rational function method is improved and the extended method is used to obtain complexiton solutions to some nonlinear differential equations.



## 1. Introduction

It is important to seek more exact traveling wave solutions of nonlinear differential equations of mathematics physics. A vast variety of powerful and direct methods to find various kinds of analytical solutions of nonlinear differential equations have been developed, which include the tanh function method [1], the sech-function method [2], the homogeneous balance method [3], the extended tanh function method [4–7], the tanh-coth method [8] and F-expansion method [9]. Recently, a direct and systematical approach, namely the transformed rational function method, was presented to seek to exact solutions of nonlinear equations by using rational function transformations in [10]. The method is very suitable for an easier and more effective handling of the solution process of nonlinear equations, unifying the existing solution methods mentioned above. Its key point is to find rational solutions to variable-coefficient ordinary differential equation transformed from given nonlinear partial differential equation. In [10], this method was applied to the $(3+1)$-dimensional Jimbo–Miwa equation and some exact traveling wave solutions, which include those solutions obtained by other methods.

In [11,12], a novel class of explicit exact solutions to the Korteweg–de Vries equation was presented through its bilinear form. Such solutions possess singularities of combinations of trigonometric function waves and exponential function waves which have different traveling speeds of new type. It was named complexiton solutions in the literature. This type of solutions would help us in recognizing a great diversity of motions of nonlinear waves described by soliton equations.

In this paper, we would like to improve the transformed rational function method so that this method can be used to seek complexiton solutions to nonlinear equations.

This paper is arranged as follows. In Section 2, an extended transformed rational function method is proposed. In Section 3, some applications are illustrated. Finally, in Section 4, some conclusions are given.

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## 2. Extended transformed rational function method

The transformed rational function method was presented in Ref.[10]. This method was used to seek traveling wave solutions of nonlinear equations. The general steps can be briefly described as follows.

Let us begin with a given partial differential equation

$$P(u, u_x, u_t, u_{xx}, \dots) = 0. \tag{1}$$

Step 1: We seek traveling wave solutions of Eq. (1) in the following form:

$$u = u(\xi), \quad \xi = k(x - ct), \tag{2}$$

where $k$ and $c$ are real constants to be determined. Under the transformation (2), Eq. (1) becomes an ordinary differential equation

$$P(u, ku', -kcu', k^2u'', \dots) = 0, \tag{3}$$

where $u' = \frac{du}{d\xi}$.

Step 2: We search for traveling wave solutions determined by

$$u^{(r)}(\xi) = v(\eta) = \frac{p(\eta)}{q(\eta)} = \frac{p_m\eta^m + p_{m-1}\eta^{(m-1)} + \cdots + p_0}{q_n\eta^n + q_{n-1}\eta^{(n-1)} + \cdots + q_0}, \tag{4}$$

where $p(\eta)$ and $q(\eta)$ are polynomials, $r > 0$ represents the minimal differential number in (3).

An important step in the solution process is to introduce a new variable $\eta = \eta(\xi)$ by a solvable ordinary differential equation, for example, a first-order differential equation:

$$\eta' = T = T(\xi, \eta), \tag{5}$$

where T is a function of $\xi$ and $\eta$, and the prime denotes the derivative with respect to $\xi$.

Thus, we obtain

$$\frac{du^{(r)}(\xi)}{d\xi} = T\frac{dv}{d\eta}, \quad \frac{du^{(r+1)}(\xi)}{d\xi} = T^2\frac{d^2v}{d\eta^2} + T'\frac{dv}{d\eta}, \dots \tag{6}$$

Then we just need to force the numerator of the resulting rational function in the transformed equation to be zero. This yields a system of algebraic equations.

Step 3: We may obtain traveling wave solutions to Eq. (1) after solving the above mentioned algebraic equations in Step 2.

In Ref.[10], it is showed that the transformed rational function method will be the exp-function method if we choose $\eta' = \eta$ and $\eta = e^{\xi}$ and that the transformed rational function method will be the extended tanh-function method if we choose $\eta' = \alpha + \eta^2$, where $\alpha$ is a constant. It is obvious that the transformed rational function method unifies the existing methods using tanh-function type functions, tan-function type functions and the exponential functions.

However, it is not appropriate to construct complexiton solutions to nonlinear equations, since complexiton solutions have different traveling wave speeds of new type. In order to obtain complexiton solutions, we improve the transformed rational function method as follows.

Let us talk about a given partial differential equation (1).

Step 1: Suppose Eq. (1) has a Hirota bilinear form:

$$H(D_x, D_t, \dots)f \cdot f = 0, \tag{7}$$

where $D_x, D_t, \dots$, are Hirota's differential operators defined by

$$D_y^p f(y) \cdot g(y) = (\partial_y - \partial_{y'})^p f(y)g(y')|_{y'=y} = \partial_{y'}^p f(y + y')g(y - y')|_{y'=0}, \quad p \geqslant 1. \tag{8}$$

Step 2: Suppose

$$f = \frac{p(\eta_1, \eta_2)}{q(\eta_1, \eta_2)}, \tag{9}$$

where $p(\eta_1, \eta_2)$ and $q(\eta_1, \eta_2)$ are polynomials and $\eta_1$ and $\eta_2$ admit, for example,

$$\eta_1'' = \frac{d^2\eta_1}{d\xi_1^2} = -\eta_1, \tag{10}$$

$$\eta_2'' = \frac{d^2\eta_2}{d\xi_2^2} = \eta_2, \tag{11}$$