Applied Mathematics and Computation 237 (2014) 168–175

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical solution for three-dimensional nonlinear mixed Volterra–Fredholm integral equations via three-dimensional block-pulse functions

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ARTICLE INFO

Keywords:
Nonlinear mixed Volterra–Fredholm integral equations
Block-pulse functions
Operational matrices
Orthogonal functions
Numerical solution

ABSTRACT

We are concerned here with a three-dimensional nonlinear mixed Volterra–Fredholm integral equations of the second kind which include many key integral that appear in the theory of nonlinear parabolic boundary value problems. The existence of a unique solution will be proved. A new numerical method for solving these type of equations will be presented. The method is based upon three-dimensional block-pulse functions approximation. In addition convergence analysis of the method is discussed. Illustrative examples are included to demonstrate the validity and applicability of the technique.

1. Introduction

Let us consider the general three-dimensional nonlinear mixed Volterra–Fredholm integral equations of the second kind of the form

$$u(x,y,z) = f(x,y,z) + \int_0^x \left(\int\int_\Omega H(x,y,z,s,t,r,u(s,t,r))drdt \right) ds, \quad (x,y,z) \in [0,1) \times \Omega, \tag{1}$$

where $u(x,y,z)$ is an unknown function, $f(x,y,z)$ and $H(x,y,z,s,t,r,u(s,t,r))$ are analytical functions on $[0,1)\times\Omega$ and $([0,1)\times\Omega)^2 \times ₵$, respectively and Ω is close subset on R^2. The existence and uniqueness of the solution for the two-dimensional mode of problem (1) are discussed in [1,2]. Equations of this type arise in the theory of nonlinear parabolic boundary value problems, the mathematical model of the spatiotemporal development of an epidemic an various physical, mechanical, and biological problems [3,4]. In recent years significant progress has been made in numerical analysis of linear two-dimensional mixed Volterra–Fredholm integral equations [5]. Fewer numerical methods are known for the nonlinear integral equations and especially for two-dimensional models [6,7]. Recently, the time collocation and time discretization method [8], the particular trapezoidal Nyström method [9] and the Adomian decomposition method [10,11] are applied for solving two dimensional linear and nonlinear integral equations. In the following we assume that

$$H(x,y,z,s,t,r,u(s,t,r)) = k(x,y,z,s,t,r)[u(s,t,r)]^p, \tag{2}$$

where p is positive integer. In the present paper, we apply three-dimensional block-pulse functions (3D-BFs), to solve the three-dimensional nonlinear mixed Volterra–Fredholm integral Eq. (1) with (2).

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http://dx.doi.org/10.1016/j.amc.2014.03.087
0096-3003/

1.1. Existence and uniqueness theorem

We consider problem (1) on the complete metric space of complex-valued continuous functions

$$X = (C(S), d); \quad d(f,g) = \sup\{|f(x,y,z) - g(x,y,z)| : (x,y,z) \in S\},$$

where $S = [0,1] \times [0,1] \times [0,1]$.

Theorem 1. *Let f and H be continuous functions on S and $S \times S \times ₵$ respectively. If*

$$|H(x,y,z,s,t,r,u(s,t,r)) - H(x,y,z,s,t,r,v(s,t,r))| \leqslant L|u(s,t,r) - v(s,t,r)|,$$

for a nonnegative constant $L \leqslant 1$, then problem (1) has a unique solution.

Proof. For the proof, see [1,2]. □

2. 3D-BFs and their properties

2.1. Definition

The m^3-set of 3D-BFs consists of m^3 functions which are defined over district $D = [0,1) \times [0,1) \times [0,1)$ as follows:

$$\phi_{i,j,l}(x,y,z) = \begin{cases} 1, & (i-1)h \leqslant x < ih, (j-1)h \leqslant y < jh, (l-1)h \leqslant z < lh \\ 0, & \text{otherwise} \end{cases}, \quad i,j,l = 1(1)m,$$

where m is positive integer, and $h = \frac{1}{m}$. Since each 3D-BFs takes only one value in its subregion, the 3D-BFs can be expressed by the three one-dimensional block-pulse functions (1D-BFs):

$$\phi_{i,j,l}(x,y,z) = \phi_i(x)\phi_j(y)\phi_l(z), \tag{3}$$

where $\phi_i(x),\ \phi_j(y)$ and $\phi_l(z)$ are the 1D-BFs related to the variables $x,\ y$ and z, respectively. The 3D-BFs are disjointed with each other:

$$\phi_{i,j,l}(x,y,z)\phi_{i',j',l'}(x,y,z) = \begin{cases} \phi_{i,j,l}(x,y,z), & i = i',\ j = j',\ l = l' \\ 0, & \text{otherwise} \end{cases}, \tag{4}$$

and are orthogonal with each other:

$$\int_0^1 \int_0^1 \int_0^1 \phi_{i,j,l}(x,y,z)\phi_{i',j',l'}(x,y,z)dzdydx = \begin{cases} h^3, & i = i',\ j = j',\ l = l' \\ 0, & \text{otherwise} \end{cases},$$

where $x,y,z \in [0,1)$ and $i,j,l,i',j',l' = 1(1)m$.

2.2. Vector forms

Now, consider the first m^3 terms of 3D-BFs and write them concisely as m^3-vector:

$$\Phi(x,y,z) = [\phi_{1,1,1}(x,y,z), \ldots, \phi_{1,1,m}(x,y,z), \ldots, \phi_{1,m,m}(x,y,z), \ldots, \phi_{m,m,m}(x,y,z)]^T, \tag{5}$$

where $x,y,z \in [0,1)$. From (4), we have:

$$\Phi(x,y,z)\Phi^T(x,y,z) = diag(\Phi(x,y,z)). \tag{6}$$

Let X be a m^3-vector by using (6) we will have:

$$\Phi(x,y,z)\Phi^T(x,y,z)X = \widetilde{X}\Phi(x,y,z),$$

where $\widetilde{X} = diag(X)$ is a $m^3 \times m^3$ diagonal matrix.

2.3. 3D-BFs expansions

A function $f(x,y,z)$ defined over district $L^2(D)$ may be expanded by the 3D-BFs as:

$$f(x,y,z) \simeq \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{l=1}^{m} f_{i,j,l}\phi_{i,j,l}(x,y,z) = F^T\Phi(x,y,z) = \Phi^T(x,y,z)F,$$