



# Cross-diffusion induced stationary patterns in a prey–predator system with parental care for predators <sup>☆</sup>



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## ARTICLE INFO

### Keywords:

Predator–prey model  
Cross-diffusion  
Stationary patterns  
Stable

## ABSTRACT

This paper is concerned about a strongly-coupled nonlinear reaction–diffusion system, which describes a prey–predator model with parental care for predators. The long time behaviors of the solution are discussed, stability and instability of the positive constant equilibrium are studied. Our results show that even though the unique positive constant steady-state is stable for the kinetic system and for the self-diffusion reaction system, cross-diffusion can generate the stationary patterns (nonconstant positive steady states). The numerical simulation is also given to illustrate the theoretical result.

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## 1. Introduction

Patterns are the heterogeneous macrostructure with certain temporal or spatial orderliness, and it exists universally in nature. Various of pattern structures constitute a colorful world. Therefore, making out the causes and mechanisms of patterns formation has a very important theoretical significance for the understanding of the mystery of nature formed.

Two different mechanisms which lead to pattern formation are known [8]: self-assembly and self-organization. Self-assembly is typical of spontaneous processes tending towards equilibrium. It is associated with the minimization of a variational energy functional in a closed system and the resulting pattern can survive indefinitely without external energy input. On the other hand self-organization implies a far from equilibrium process, and is possible only in open system with an external energy source. Prototype models of self-organization generated patterns are the reaction–diffusion systems that, since the seminal paper of Turing [18], have attracted a growing interest as they constitute an essential basis to describe morphogenetic mechanisms. Turing [18] showed that diffusion could destabilize an otherwise stable equilibrium of the reaction–diffusion system and lead to nonuniform spatial patterns. This kind of instability is usually called Turing instability or diffusion-driven instability.

Patterns formation in the reaction–diffusion predator–prey systems has been studied in ecology for more than thirty years. Starting with the pioneering work of Segel and Jackson [17], they applied Turing’s reaction–diffusion to population dynamics and found a way to explain the mechanism from a more intuitive perspective than had previously been done. Several researches have formulated conditions for the occurrence of Turing instability in specific systems. Murray [10] detailed spatial patterns formation with reaction–diffusion systems and gave many examples to explain it. Bartumeus et al. [4] demonstrated that a simple reaction–diffusion predator–prey model with a ratio-dependent functional response for the predator can develop diffusion driven instabilities known as Turing structures by using linear stability analysis. Alonso et al. [1]

<sup>☆</sup> This work is partially supported by NSFC under Grant No. 11371311 and also by the Ph.D. Programs Foundation of Ministry of Education of China under Grant No. 20113250110005.

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obtained that mutual interference between predators can generate Turing structures in a homogeneous environment under certain conditions of trophic interaction and predator–prey relative diffusion.

However, most of the reaction–diffusion systems used to predict the occurrence of patterns with self-diffusion terms and omitted the cross-diffusion terms (together with self-diffusion terms). Therefore, self-diffusion and cross-diffusion terms are introduced in the context of population dynamics and gained a renewed interest [6,19,20,24]. Hence, how cross-diffusion destabilizes uniform equilibrium is investigated in this paper.

In the classical predator–prey model that each individual predator admits the same ability to attack prey. This assumption is obviously unrealistic for many animals. In the natural world, there are many species whose individuals have a life history that take them through two stages, immature and mature, where immature predators are raised by their parents. Therefore, stage-structured population models also have received great attention in recent years. Aiello and Freedman [3] studied a single-species growth model with stage structure consisting of immature and mature stages by using a discrete time delay. Cui et al. [5] analyzed the asymptotic behavior of some autonomous and time-varying population models with stage-structure. Wang and Chen [21] considered the asymptotic behavior of a predator–prey model with stage structure and found that an orbitally asymptotically stable periodic orbit exists. Song and Chen [16] obtained conditions for global asymptotic stability of three nonnegative equilibria and a threshold of harvesting for the mature prey population. Shi and Chen [15] proposed a ratio-dependent predator–prey model with stage structure in the prey and obtained sufficient conditions for the existence and stability of equilibriums. Agarwal and Devi [2] dealt with a ratio-dependent predator–prey–resource model with stage structure and obtained the positivity, stability, bifurcation and persistence of that model. However, most of them assume that the transition rate from the immature stage to the mature stage is a constant. This implies that immature predators have enough food to live on at any time. This could be the case where immature predators feed on a resource which is different from the prey and is very rich. If the food of immature predators is given by their parents (adult predators) or is obtained by themselves through attacking the prey, the transition rate is clearly affected by the nutrient availability, and this may produce rich dynamical behaviors of prey–predator interactions. Therefore, we incorporate the nutrient intake rate of immature predators into the transition rate and consider its influences on the dynamics of the prey–predator interactions. It may result in the emergence of stationary patterns.

For this purpose, we choose a stage-structure predator–prey model [23] with parental care for our studies:

$$\begin{cases} \frac{du}{dt} = u(r - \alpha u) - \beta u v_2, \\ \frac{dv_1}{dt} = k_1 \beta u v_2 \frac{v_2}{w v_1 + v_2} - m_1 v_1 - k_2 \beta u v_2 \frac{w v_1}{w v_1 + v_2}, \\ \frac{dv_2}{dt} = k_2 \beta u v_2 \frac{w v_1}{w v_1 + v_2} - m_2 v_2, \end{cases} \tag{1.1}$$

where  $u$ ,  $v_1$  and  $v_2$  denote the densities of prey, immature predator and mature predator, respectively.  $r$  is the intrinsic growth rate of the prey.  $\alpha$  is the density dependent coefficient of the prey.  $\beta$  is the attacking rate of the mature predator at the prey.  $w$  measures the relative consumption ratio between one immature predator and one mature predator.  $k_1$  is a conversion coefficient of the immature predator.  $k_2$  is a proportional constant and this kind of transition rate was used by Roughgarden et al. [14] for an open marine population.  $m_1$  and  $m_2$  are the death of the immature predator and mature predator. We always assume that adult predators forage the prey and provide parental care to their offsprings and all the parameters in system (1.1) are assumed to be positive.

Using the scaling:

$$k_2 \beta u / m_2 \rightarrow u, w v_1 \rightarrow v_1, v_2 \rightarrow v_2, m_2 t \rightarrow t$$

and then (1.1) reduces to

$$\begin{cases} \frac{du}{dt} = u(a - bu - cv_2), \\ \frac{dv_1}{dt} = \frac{ku v_2^2}{v_1 + v_2} - m v_1 - \frac{w u v_1 v_2}{v_1 + v_2}, \\ \frac{dv_2}{dt} = \frac{u v_1 v_2}{v_1 + v_2} - v_2, \end{cases} \tag{1.2}$$

where  $a = r/m_2$ ,  $b = \alpha/(k_2 \beta)$ ,  $c = \beta/m_2$ ,  $k = w k_1/k_2$  and  $m = m_1/m_2$ .

Denote  $\mathbf{u} = (u, v_1, v_2)^T$  and

$$G(\mathbf{u}) = \begin{pmatrix} G_1(\mathbf{u}) \\ G_2(\mathbf{u}) \\ G_3(\mathbf{u}) \end{pmatrix} = \begin{pmatrix} u(a - bu - cv_2) \\ \frac{ku v_2^2}{v_1 + v_2} - m v_1 - \frac{w u v_1 v_2}{v_1 + v_2} \\ \frac{u v_1 v_2}{v_1 + v_2} - v_2 \end{pmatrix},$$

then (1.2) can be further written as

$$\frac{d\mathbf{u}}{dt} = G(\mathbf{u}). \tag{1.3}$$

It is easy to see that if and only if

$$a > b, \quad k(a - b)^2 > b[w(a - b) + bm], \tag{1.4}$$

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