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Robust exponential H_∞ control for uncertain time-varying delay systems with input saturation: A Markov jump model approach ☆

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ABSTRACT

This paper is concerned with the state feedback stabilization problem for uncertain time-varying delay systems with input saturation. Based on the stochastic property of time-varying delay, the considered system can be transformed into a continuous Markov jump system with stochastic delays, where the growth trend and the maximum value of the time-varying delay are limited. The problem we address is to design a robust state feedback controller which can switch with time-varying delay in terms of Markov process, such that the system is exponentially mean-square stable with disturbance attenuation γ. Conditions for the existence of solutions to this problem are obtained in terms of linear matrix equalities (LMIs). When these LMIs are feasible, the desired robust controller is given. Finally, two examples are provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

In recent years, much attention has been drawn to the problem of robust H_∞ control [10,15,18,22,23], which takes both the performance requirements and system parameter uncertainties into account. More recently, the robust H_∞ control approach has been applied into many complex dynamic systems. Among these complex systems, the time-delay systems have been a topic of recurring interest over the past decades. There are many kinds of delays in dynamic systems, such as certain delays, time-varying delays [1,8,11,24,13,36,41], switched time-delays [3], mixed time-delays [37,38], distributed time-delays [21], mode-dependent time-delays [25,28,31] and so on. Because of the instability and poor performance caused by time delays in various engineering systems, the problem of robust stability analysis and robust stabilization for uncertain time-delay systems have been studied. The achievements of criteria about time-delay systems includes two types, which are delay-independent [30] and delay-dependent results [9,12,20,32,40]. Generally speaking, delay-dependent criteria is less conservative than the delay-independent one since it can make full use of the information of the systems, especially when time delay is small. In this paper, not only the lowest and uppermost bounds of time delays are employed, but also the

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probability of time delays is investigated. So it's meaningful and necessary to study the way to design controller by taking account into the stochastic characteristics of the time delays.

On the other hand, since it is usually impossible to provide infinite energy of the signal due to the limited of the expense and technic [27], control input saturation becomes more and more common in real practical application. It's known that input saturation often leads to instability, degradation of system performance and parasitic equilibrium points of a control system. Be aware of the great importance of the system subjected to input saturation many researchers have studied this topic from practical and theoretical points of view. There are a lot of results about it and the references therein [4,6,16,17,19,29]. For instance, by using passivity theorem to deal with input saturation, some analysis results were presented in [14]. An optimization-based approach was proposed to design feedback and anti-windup gains of a controller subject to saturation in [5]. When both the input saturation and time delay appear in the systems, the problem becomes more complicated. As a class of stochastic hybrid systems, Markovian jump systems have been extensively studied in the past decades, see for example, [2,34,35,39,42,43]. We can draw on the experience of the method about Markov jump model to deal with the problem mentioned above.

In this paper, we study the stabilization for a class of uncertain time-delay systems with the constraints of control input saturation. By separating the delay interval into several subintervals, the mechanism tries to extract more effective information from the stochastic time delay. The original system has been transformed into Markovian jump time-delay systems so that a input saturation controller can be achieved. By solving a convex optimization problem with LMIs constraints, a suit memoryless state feedback robust H_∞ control law is designed. The illustrative examples are given to show the effectiveness of the proposed approach.

Notation. Throughout the paper, for symmetric matrices X and Y, the notation $X \geqslant Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The notation N^T represents the transpose of the matrix N; $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symbol $*$ is used to be an ellipsis for the terms induced by symmetry and diag$\{\cdots\}$ stands for a block-diagonal matrix, and $(M)^* \triangleq M + M^T$. If A is a symmetric matrix, $\lambda_{max}(A)$ and $\lambda_{min}(A)$ denote the largest and smallest eigenvalue of A, respectively. The space of square-integrable vector functions over $[0,\infty)$ is denote by $L_2[0,\infty)$, and for $\omega_2 = \sqrt{\int_{t=0}^{\infty} |\omega(t)|^2 dt}$.

2. Preliminaries and problem formulation

Consider the following class of uncertain linear systems with stochastic time-varying delays described as

$$\begin{aligned}
\dot{x}(t) &= A(t)x(t) + A_d(t)x(t-\tau(t)) + B_1 sat(u(t)) + B_2\omega(t),\\
y(t) &= Cx(t),\\
u(t) &= Kx(t),\\
x(t) &= \psi(t), \quad \forall t \in [-\bar{\tau}, 0]
\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^q$ is the measurement output, $u(t)$ is the control input, $\omega(t)$ is the disturbance input which belongs to $L_2[0,\infty)$. $A(t)$ and $A_d(t)$ are uncertain system parameters matrices of appropriate dimensions, and $B_1,\ B_2$ and C are constant system parameter matrices of appropriate dimensions.

The plant inputs are supposed to be bounded as follows:

$$-u_{0(i)} \leqslant sat(u_{(i)}) \leqslant u_{0(i)}, \quad u_{0(i)} > 0,\ i = 1, \ldots,\ m.$$

In system (1), we assume that

$$A(t) = A + \triangle A(t),$$

$$A_d(t) = A_d + \triangle A_d(t),$$

where $\triangle A(t)$ and $\triangle A_d(t)$ are real time-varying matrix functions representing parameter uncertainties.

Without loss of generality, the admissible parameter uncertainties in this paper are assumed to be modeled as

$$[\triangle A(t) \quad \triangle A_d(t)] = DF(t)[E_A \quad E_{A_d}], \tag{2}$$

where $D,\ E_A$ and E_{A_d} are known real constant matrices. $F(t)$ is an unknown time-varying matrix function satisfying

$$F^T(t)F(t) \leqslant I. \tag{3}$$

The time-varying delay $\tau(t)$ satisfies

$$0 \leqslant \underline{\tau} \leqslant \tau(t) \leqslant \bar{\tau}, \quad \dot{\tau}(t) \leqslant \mu. \tag{4}$$