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On the dynamics of a restricted Cournot–Puu triopoly: Firms survival and complexity

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ABSTRACT

In this paper we explore the dynamics of a restricted Cournot–Puu triopoly. We focus our interest in a triopoly when two firms have both the same marginal costs and initial conditions and explore the dynamics in this situation which was introduced in Puu (1998) [14]. We find conditions for removing the other firm from the market, and analyze the complexity of the map by means of sample permutation entropy.

1. Introduction

In 1991, Tönu Puu introduced a Cournot type duopoly model with isoelastic demand function and constant marginal costs [13]. The dynamic model is given by

$$q_1(t+1) = \max\left\{0, \sqrt{\frac{q_2(t)}{c_1}} - q_2(t)\right\},$$
$$q_2(t+1) = \max\left\{0, \sqrt{\frac{q_1(t)}{c_2}} - q_1(t)\right\},$$

where c_1 and c_2 are the constant marginal costs of both firms, and $q_1(t)$ and $q_2(t)$ are the outputs of each firm at time $t \in \mathbb{N} \cup \{0\}$. For any non-negative initial conditions $(q_1(0), q_2(0))$ we have a unique solution, called orbit or trajectory. From the dynamical systems point of view, the main aim is to understand the asymptotic behavior of all its orbits, which we often call it the dynamics of the system. Sometimes, the description of the asymptotic behavior of all the orbits is possible. For instance, it is easy to see that when $c_1 = c_2 = c$ the system has two fixed points $(0,0)$ and $(\frac{1}{4c}, \frac{1}{4c})$ and a 2-periodic orbit given by $(0, \frac{1}{4c})$ and $(\frac{1}{4c}, 0)$, and all the remaining orbits converge to one of these periodic orbits. In addition, any orbit with initial condition in $(0, \frac{1}{c})^2$ converges to $(0, \frac{1}{4c})$. However, frequently these dynamic description is not possible and therefore, we must study some so-called dynamical properties of the system, as for instance, the existence of periodic orbits, chaotic attractors and measures of complexity as topological entropy or Lyapunov exponents.

The dynamics of Cournot–Puu duopoly has been extensively studied and, except for the existence of absolutely continuous ergodic measures, its dynamics is completely known (see the survey paper [11] and the references therein). The main key for this knowledge is that we can reduce it to a one-dimensional model by noticing that if

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$$F(q_1,q_2) = (f_1(q_2), f_2(q_1)) = \left(\max\left\{0, \sqrt{q_2/c_1} - q_2\right\}, \max\left\{0, \sqrt{q_1/c_2} - q_1\right\}\right),$$

then

$$F^2(q_1,q_2) = ((f_1 \circ f_2)(q_1), (f_2 \circ f_1)(q_2))$$

and so we can study the system by analyzing one-dimensional systems given by $f_1 \circ f_2$ and $f_2 \circ f_1$. This idea was introduced first in [12] for duopoly models having the same structure as Puu's duopoly and for instance, combining the dynamics of both composition we can characterize the attractors of F with non-empty interior (see e.g., [4,11]).

If we increase the number of competitors to have a triopoly, the model can be written as

$$q_1(t+1) = \max\left\{0, \sqrt{\frac{q_2(t)+q_3(t)}{c_1}} - (q_2(t)+q_3(t))\right\},$$
$$q_2(t+1) = \max\left\{0, \sqrt{\frac{q_1(t)+q_3(t)}{c_2}} - (q_1(t)+q_3(t))\right\},$$
$$q_3(t+1) = \max\left\{0, \sqrt{\frac{q_1(t)+q_2(t)}{c_3}} - (q_1(t)+q_2(t))\right\}.$$

Now, we cannot reduce our problem to a one-dimensional one and hence, the analysis of the dynamics of this model is much more complicated than in the duopoly case. Moreover, although numerical simulations give us a rough idea of how the dynamics can be (see [1,14]), analytically just we can address some local results related to the fixed points (Cournot points) of the system [2].

Hence, in order to make some advances in the understanding of global dynamics, we can follow two different research lines. The first one is to explore some interesting economic phenomena, for instance, when the firms can disappear from the market in such a way it is reduced to a duopoly (see [10]). The second line includes some simplifications of the above system like the fact that both firms have the same marginal costs (see [14]).

The idea of this paper is combining the above research lines by studying in detail the Cournot–Puu model with two possible marginal costs. Hence, we give precise results which guarantee that one firm is removed from the market in a restricted case. On the other hand, we analyze the parameter values for which the system has a complicated dynamical behavior by using a permutation entropy type measure. Finally, we will make an approach to the oligopoly case when the number of firms is greater than 3.

2. Statement of the model: first properties

Consider an oligopoly of n firms with isoelastic demand function (in its inverse form)

$$p = \frac{1}{\sum_{i=1}^{n} q_i} = \frac{1}{Q}, \quad (1)$$

where p is the price, $q_i, i = 1,\ldots,n$, is the output of each firm and Q is the total production. Assume also costs functions given by

$$C_i(q_i) = c_i q_i, \quad i = 1,\ldots,n, \quad (2)$$

where c_i is constant for $i = 1,\ldots,n$. Maximizing the profit functions

$$\Pi_i(q_1,\ldots,q_n) = \frac{q_i}{Q} - c_i q_i, \quad i = 1,\ldots,n, \quad (3)$$

we obtain the reaction functions

$$q_i = f_i(q_1,\ldots,q_n) = \sqrt{\frac{Q_i}{c_i}} - Q_i, \quad i = 1,\ldots,n, \quad (4)$$

where $Q_i = Q - q_i$ is the residual supply of each firm. If we make the game dynamic and assume that firms have naive expectations, we have that each firm reacts following the maximal profit rule

$$q_i(t+1) = f_i(q_1(t),\ldots,q_n(t)) = \max\left\{0, \sqrt{\frac{Q_i(t)}{c_i}} - Q_i(t)\right\}, \quad i = 1,\ldots,n \quad (5)$$

for $t \geqslant 0$. Note that outputs (productions) cannot be negative. Non null fixed points of the model are usually called Cournot points. The stability of such points is very important in oligopoly dynamics. Notice that the max function makes the maps f_i to be continuous but they are non-differentiable at the frontier of its support. Recall that for a non-negative real map $\phi : S \subset \mathbb{R}^n \to [0,+\infty)$, define its support, $\text{supp}(\phi)$, as the points x such that $\phi(x) > 0$.