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Robustness analysis for parameter matrices of global exponential stability time varying delay systems

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ABSTRACT

This paper analyzes the robustness of global exponential stability time varying delay systems subjected to parameter uncertainty. Given a globally exponentially stable time varying delay systems, the problem to be addressed herein is how much parameter uncertainty the systems can withstand to be globally exponentially stable. We characterize the upper bounds of the parameter uncertainty for the systems to sustain global exponential stability. Moreover, we prove theoretically that, for globally exponentially stable systems, if additive parameter uncertainty is smaller than the derived upper bounds arrived at here, then the perturbed systems are guaranteed to also be globally exponentially stable. A numerical example is provided to illustrate the theoretical result.

1. Introduction

Time delays are often encountered in various practical systems such as chemical processes, neural networks and long transmission lines in pneumatic systems [1,2]. It has been shown that the existence of time-delays may lead to oscillation, divergence, instability, greatly increase the difficulty of stability analysis and control design. Many researchers in the field of control theory and engineering study the robust control of time-delay systems. The main methods of stability analysis can be classified into two types: frequency-domain and time-domain. The former use the sum of squares technique. As to the time-domain approach, Lyapunov functional is a powerful tool, which can deal with time varying delays.

For most successful applications of the systems, the stability is usually a prerequisite. The stability of the systems depends mainly on their parametrical configuration. Moreover, in the applications of the systems, external random disturbances and time delays are common and hardly avoided. It is known that random disturbances and time delays in the systems may result in oscillation or instability of the nonlinear systems. The stability analysis of the delayed systems and the systems with external random disturbances has been widely investigated in recent years (see, e.g., [3–19], and the references cited therein).

In practice, when we estimate systems parameter matrices, there are always some uncertainty and errors. If the uncertainty are too large, then the stable systems may be instable, the intensity of parameter matrices uncertainty is often leaded to instability and they can destabilize stable delay systems if it exceeds its limits. The instability depends on the intensity of parameter matrices uncertainty. Many people analysis the robust stability of parameter uncertainty systems [20–34]. For

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stable delay systems, if the intensity of parameter matrices uncertainty is low, the stable delay systems may still be stable. Therefore, it is interesting to determine how much parameter matrices uncertainty of stable delay systems can withstand without losing global exponential stability. Although the various stability properties of stable delay systems have been extensively investigated using the Lyapunov and the linear matrix inequality methods, the robustness of the global stability for parameter matrices of systems is rarely analyzed directly by estimating the upper bounds of parameter matrices uncertainty level.

Motivated by the above discussions, our purpose is to quantify the parameter uncertainty level for stable delay systems in this paper. Compared with the conventional Lyapunov stability theory and linear matrix inequality methods, we investigate the robust stability for global exponential stability directly from the coefficients of the delay systems which should satisfy the global exponential stability condition. For the parameter matrices uncertainty, two types are studied widely: time varying structured uncertainty and polytopic-type uncertainty. In this paper, we characterize the robustness for parameter matrices in general form of stable delay systems by deriving the upper bounds of parameter matrices uncertainty for global exponential stability. We prove theoretically that, for globally exponentially stable delay systems, if additive parameter matrices uncertainty are smaller than the derived upper bounds herein, then the stable delay systems are guaranteed to be globally exponentially stable.

Notations: Throughout this paper, unless otherwise specified, R^n and $R^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of $n\times m$ real matrices. If A is a matrix, its operator norm is denoted by $\|A\|=\sup\{|Ax| : |x|=1\}$, where $|\cdot|$ is the Euclidean norm. If A and B are two symmetric matrix, then $A>B$ iff $A-B>0$. $C([-\bar{\tau},0];R^n)$ denotes the space of all continuous R^n- valued functions φ defined on $[-\bar{\tau},0]$ with a norm $||\varphi||=\sup_{-\bar{\tau}\leqslant\theta\leqslant 0}|\varphi(\theta)|$.

2. Problem formulation

Consider the following systems with time-varying delay

$$\dot{x}(t)=Ax(t)+Bx(t-\tau(t)) \tag{1}$$

$$x(t)=\psi(t-t_0),\ t_0-\bar{\tau}\leqslant t\leqslant t_0, \tag{2}$$

where $x(t)=(x_1(t),\dots,x_n(t))^T\in R^n$ is the state vector of the systems, $t_0\in R_+$ and $\psi\in R^n$ are the initial values, $A\in R^{n\times n}$, $B\in R^{n\times n}$ are parameter matrices, $\tau(t)$ is a time varying delay that satisfies $\tau(t):[t_0,+\infty)\to[0,\bar{\tau}]$, $\tau'(t)\leqslant\mu<1$, $\psi=\{\psi(s):-\bar{\tau}\leqslant s\leqslant 0\}\in C([-\bar{\tau},0],R^n)$, $\bar{\tau}$ is the maximum of delay, Now we define the global exponential stability of the state of systems (1) and (2).

Definition 1. The state of systems (1) and (2) is globally exponentially stable, if for any t_0, ψ, there exist $\alpha>0$ and $\beta>0$ such that

$$|x(t;t_0,\psi)|\leqslant\alpha||\psi||\exp(-\beta(t-t_0)),\quad\forall t\geqslant t_0, \tag{3}$$

where $x(t;t_0,\psi)$ is the state of the systems (1) and (2).

Numerous criteria for ascertaining the global exponential stability of systems (1) and (2) have been developed; e.g., [18,21,22,28,30,31] and the references therein.

Now, the question is given a globally exponentially stable systems (1) and (2), how much the parameter uncertainty matrices the systems (1) and (2) can bear? The new systems (1) and (2) are changed as

$$\dot{y}(t)=(A+\Delta A)y(t)+(B+\Delta B)y(t-\tau(t)), \tag{4}$$

$$y(t)=\psi(t-t_0),\quad t_0-\bar{\tau}\leqslant t\leqslant t_0. \tag{5}$$

The parameter matrices structured uncertainties are assumed to be of the form

$$[\Delta A,\Delta B]=[A_1F_1(t)A_2,B_1F_2(t)B_2] \tag{6}$$

where A_1, A_2, B_1, B_2 are constant matrices with appropriate dimensions; and $F_i(t)$ are unknown, real, and possibly time-varying matrices with Lebesgue-measurable elements satisfying

$$F_i^T(t)F_i(t)\leqslant I,\quad\forall t,\ i=1,2. \tag{7}$$

For the polytopic-type uncertainty, that is, the uncertain matrices ΔA, ΔB satisfy the real, convex, polytopic-type model

$$[\Delta A,\Delta B]\in\Omega,\Omega=\left\{[A(\xi),B(\xi)]=\sum_{j=1}^{k}\lambda_j[A_j,B_j],\sum_{j=1}^{k}\lambda_j=1,\lambda_j\geqslant 0\right\} \tag{8}$$

where A_j, B_j are known matrices and called the vertices of the polytope.