Applied Mathematics and Computation 237 (2014) 411–418

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Multi-choice linear programming for matrix game

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ARTICLE INFO

Keywords:
Linear programming
Mixed integer programming
Multi-choice programming
Matrix game
Transformation technique

ABSTRACT

The aim of this paper is to develop a multi-choice model for matrix game problem where a payoff matrix is multi-choice in nature. Then it transfer to a standard mathematical programming problem such a way of alternatives, out of which one is to be selected. The selection of alternatives should be the combination of choices provides an optimal solution to the matrix game. There may be more than one combination which will provide an optimal solution. However, the problem cannot be solved by standard linear programming problem (LPP) techniques. This paper proposed a technique to formulate mixed-integer programming model. Using the standard soft ware, the proposed model can be solved. Finally numerical example is presented to illustrate the proposed model and solution procedure.

1. Introduction

Game theory has a remarkable importance in both Operations Research and Systems Engineering due to its great applicability. Many real conflict problems can be modeled as games. However, the encountered conflict problems in military, economical and political fields become more and more complex and uncertain due to the existence of diversified factors. This situation will bring some difficulties in application of classical game theory. To remove this difficulties, we have employed multi-choice options to two-person zero-sum game.

In this paper, few references are presented including their work. The vector payoffs were first considered by Backwell [6] and later by Contini. Both writers are formalized the problem with its full of stochastic information. Fernandez et al. [11] considered to solve the two-person multicriteria zero-sum games. As they have considered a multicriteria game, the solution concept is based on Pareto optimality and finally they obtained the Pareto efficient solution for their proposed games. Fernandez and Puerto [10] have developed a methodology to get the whole set of Pareto-optimal security strategies which are based on solving a multiple criteria linear program. This approach shows the parallelism between these strategies in multicriteria games and minimax strategies in scalar zero-sum matrix games. This notion of security is based on expected payoffs. For this reason, only as the game is played many times these strategies provide us a real sense of security. In the contrary, if the game is played only once; as in one shot games, a better analysis should be considered not only the payoffs but also the probability to get them. Ghose and Prasad [7], have been proposed to a solution concept based on Pareto-optimal security strategies for these games. They also introduced the concept based on the similarity with security levels are determined by the saddle points in scalar matrix games. This concept is independent of the notion of equilibrium so that the opponent is to be taken into the account to establish the security levels for one's own payoffs. When it is used to select strategies, the concept of security level has important property which the payoff obtained by these strategies cannot be diminished by

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http://dx.doi.org/10.1016/j.amc.2014.03.017
0096-3003/

the opponent's deviation in strategy. Roy [18], has presented to the study of two different solution procedures for the two-persons bimatrix games. The first solution procedure is applied to the game on getting the probability to achieve some specified goals along with the player's strategy. The second specified goals along with the player's strategy for defining the fuzzy membership function defined on the pay-off matrix of the bimatrix game. Das and Roy [3], have proposed to a new solution concept by considering the entropy function to the objectives of the player. These models are known as entropy optimization models on two-person zero-sum game. Solution concept is based on the Kuhn Tucker conditions, Maximum Entropy Principle [8], and Minimum Cross-Entropy Principle [12]. Without considering the pay-off functions, they have been shown that the optimal strategy and the value of the game for each player are equivalent to the results of classical game.

Wang et al. [19] studied the outcome of the public goods game on two interdependent networks that are connected by means of a utility function, which determines how payoffs on both networks jointly influence the success of players in each individual network. They showed that an unbiased coupling allows the spontaneous emergence of interdependent network reciprocity, which is capable to maintain healthy levels of public cooperation even in extremely adverse conditions. In another paper [20] they have studied the impact of population density on the evolution of public cooperation in structured populations, and find that the optimal density is uniquely related to the percolation threshold of the host graph irrespective of its topological details. They have explained their observations by showing that spatial reciprocity peaks in the vicinity of the percolation threshold, when the emergence of a giant cooperative cluster is hindered neither by vacancy nor by invading defectors, thus discovering an intuitive yet universal law that links the population density with social prosperity. Perc et al. [13] have introduced "Collective behavior and evolutionary games". Wang et al. [21] have studied the evolution of operation in 2-person evolutionary games on networks. They also analyzed the microscopic mechanisms that give rise to the observed macroscopic behaviors in both homogeneous and heterogeneous network when a mechanism for social punishment is introduced. Perca and Szolnokib [14] have reviewed recent works on evolutionary games incorporating co-evolutionary rules, as well as give a didactic description of potential pitfalls and misconceptions associated with the subject. Wang et al. [22] studied the evolution of public cooperation on two interdependent networks that are connected by means of a utility function, which determines to what extent payoffs in one network influence the success of players in the other network. Perc et al. [15] highlighted that the study of the dynamics of group interactions, like several other important equilibrium and non-equilibrium dynamical processes in biological, economical and social sciences, benefits from the synergy between statistical physics, network science and evolutionary game theory. Perc and Wang [16] discussed on Heterogeneous Aspirations Promote Cooperation in the Prisoner's Dilemma Game. In their study, they indicated that heterogeneity in aspirations may be key for the attainability of cooperation in structured populations. Wang et al. [23] showed that in fact only an intermediate density of sufficiently strong interactions between networks warrants an optimal resolution of social dilemmas and this is due to an intricate interplay between the heterogeneity that causes an asymmetric strategy flow because of the additional links between the networks, and the independent formation of cooperative patterns on each individual network.

Multi-choice linear programming problems are existed in many managerial decision making problems. Hiller and Lieberman [9] and Ravindran et al. [2] have considered a mathematical model in which an appropriate constraint is to be chosen using binary variables required for a constraint is same as the total number of choices for the constraint. Chang [4] has proposed, a new idea for modeling the multi-choice goal programming problem. He used multiplicative terms of binary variables to handle the multiple aspiration levels. In his another paper [5], he revises multiplicative terms of the binary variables by a continuous variable. Depending on this idea Panda et al. [1] have solved multiple pay off game problem. Biswal and Acharya [17] has presented a transformation model for multi-choice linear programming in requirement vector. The present multi-choice linear programming problem can not be solved by traditional LPP techniques. In order to solve the present problem, this paper proposes a new methodology to solve matrix game problem.

2. Mathematical model

A payoff matrix of the players PI and PII is defined as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \tag{2.1}$$

The mixed strategy of the matrix game (2.1) for player PI and PII are defined as follows:

$$X = \left\{ x \in R^m;\ \sum_{i=1}^{m} x_i = 1;\ x_i \geqslant 0,\ i = 1, 2, \dots, m \right\} \tag{2.2}$$

$$Y = \left\{ y \in R^n;\ \sum_{j=1}^{n} y_j = 1;\ y_j \geqslant 0,\ j = 1, 2, \dots, n \right\} \tag{2.3}$$