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Improved delay-partitioning method to stability analysis for neural networks with discrete and distributed time-varying delays

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ABSTRACT

In this paper, an improved method is derived for the delay-dependent stability problem of neural networks with discrete and distributed time-varying delays. An improved Lyapunov functional is constructed by introducing the newly delay-partitioning method and considering the sufficient information of neuron activation functions. By using the relationship between each subinterval and time-varying delay sufficiently, a new delay-dependent stability criterion has been obtained to reduce the conservatism. Two numerical examples are finally given to show the merits of the derived conditions.

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1. Introduction

Recently, neural networks (NNs) have been successfully applied in various fields including pattern recognition, control, associative memories, chemical processes. Meanwhile, since time delays are inevitably encountered in NNs which is usually a main source of oscillation and instability, the stability of delayed NNs become an important issue. There exist many interesting stability results for delayed NNs, including delay-independent [1–3], and delay-dependent [4–45]. The delay-independent stability conditions are usually more conservative than delay-dependent conditions due to they include less information concerning the time delays, especially for the time delays are relatively small. Since the Lyapunov–Krasovskii functional (LKF) approach can reduce the conservatism effectively, so choice an suitable LKF is important for obtaining less conservative stability results. Thus, many new improved methods, such as free-weighting matrix [5–7], augmented LKF [8,9], delay-slope-dependent method [10] have been developed to reduce the conservatism in recent years. Obviously, by using the identical LKFs, it is hard to reduce the conservatism. Therefore, delay-partitioning method which divides delay interval into some subintervals is developed to reduce the conservatism effectively [12–20]. In [12–14], the delay-partition number has been chosen as two. For example, by utilizing different free-weighting matrices in each delay subintervals, a new stability criterion is proposed in [12] for NNs with time-varying delays. In [15–20], a generalized delay-partitioning method for delayed neural networks were proposed to reduce the conservatism effectively. For example, in [15], a new stability result for delayed Hopfield NNs was proposed by decomposing the delay interval into some subintervals with the same size. Very recently, a new delay-decomposing approach to delay-dependent stability for delayed NNs is introduced in [19] by considering independent upper bounds of the delay derivative in each subinterval. By use of delay-decomposing approach and reciprocally convex technique, some improved stability criteria for NNs with time-varying delays have been developed in

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[20]. However, these results have common shortcomings. On the one hand, the relationship between time-varying delay and each subinterval is completely neglected. On the other hand, some important information concerning with neuron activation functions are not adequately considered. Thus, this motivate our further investigation.

In our paper, for any integer $m \geqslant 1$, define $h = \frac{d}{m},\ \rho(t) = \frac{d(t)}{m}$, then the delay interval $[0, d]$ is decomposed into m segments, i.e., $[0, d] = \cup_{i=1}^{m}[(i-1)h, ih]$ and $\rho(t)$ satisfies $0 \leqslant \rho(t) \leqslant h$. But different from some existing methods [19,20], for each subinterval $[(k-1)h, kh], k = 1, 2, \cdots, m$, we use two different delay-partitioning methods to consider the relationship between time-varying delay and each subinterval sufficiently. Firstly, we introduced an variable $(k-1)h + \rho(t)$ and it is easy to obtain $(k-1)h + \rho(t) \in [(k-1)h, kh]$, then each subinterval $[(k-1)h, kh], k = 1, 2, \ldots, m$ is decomposed into 2 segments, i.e., $[(k-1)h, kh] = [(k-1)h,\ (k-1)h + \rho(t)] \cup [(k-1)h + \rho(t),\ kh]$. Secondly, for any $t \geqslant 0$, there exist an integer $k \in \{1, 2, \ldots, m\}$, such that $d(t) \in [(k-1)h, kh]$, then each subinterval $[(k-1)h, kh], k = 1, 2, \ldots, m$ is decomposed into another 2 segments, i.e., $[(k-1)h, kh] = [(k-1)h,\ d(t)] \cup [d(t),\ kh]$. Therefore, when handling with $\dot{V}(z_t)$, the subinterval $[(k-1)h, kh]$ is not only decomposed into two segments, i.e., $[(k-1)h,\ (k-1)h + \rho(t)], [(k-1)h + \rho(t),\ kh]$, but also decomposed into another two segments, i.e., $[(k-1)h,\ d(t)], [d(t),\ kh]$. So the relationship between time-varying delay $d(t)$ and each subinterval $[(k-1)h, kh], k = 1, 2, \ldots, m$ is further considered, which may lead to less conservative results. An augmented LKF is introduced by considering the information of activation functions sufficiently. Then, inspired by reciprocally convex approach [47], a less conservative result guaranteeing the asymptotically stable of delayed NNs is obtained. Two numerical examples are finally included to show the merits of the proposed results.

Notation: Throughout this paper, $\mathscr{R}^n$ denotes the n-dimensional Euclidean space, $\mathscr{R}^{m\times n}$ denotes the set of $m \times n$ real matrix. $\mathrm{diag}(\cdots)$ denotes the block diagonal matrix. X_{ij} denotes the element in row i and column j of matrix X. $*$ denotes the elements below the main diagonal of asymmetric matrix.

2. Problem statement and preliminaries

Consider the following recurrent delayed NNs:

$$\dot{x}(t) = -Cx(t) + W_0 g(x(t)) + W_1 g(x(t - d(t))) + D\int_{t-\tau(t)}^{t} g(x(s))\mathrm{d}s + b, \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathscr{R}^n$ is the neuron state vector. $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)), \ldots, g_n(x_n(\cdot))]^T \in \mathscr{R}^n$ denotes the neuron activation function. $W_0 \in \mathscr{R}^{n\times n}$ is the interconnection weight matrix and $W_1,\ D \in \mathscr{R}^{n\times n}$ are the delayed interconnection weight matrices. $C = \mathrm{diag}(C_1, C_2, \ldots, C_n)$ satisfies with $C_i > 0, i = 1, 2, \ldots, n$. $b = (b_1, b_2, \ldots, b_n)^T \in \mathscr{R}^n$ is a constant input vector. $d(t),\ \tau(t)$ are time-varying delays satisfying $0 \leqslant d(t) \leqslant d, \dot{d}(t) \leqslant \mu, 0 \leqslant \tau(t) \leqslant \tau$, where $d,\ \tau$ and μ are constants. Furthermore, neuron activation functions $g_j(\cdot), j = 1, 2, \ldots, n$ satisfy

$$k_j^- \leqslant \frac{g_j(x) - g_j(y)}{x - y} \leqslant k_j^+, \quad \forall x, y \in R,\ x \neq y,\ j = 1, 2, \ldots, n, \tag{2}$$

where $k_j^-, k_j^+, j = 1, 2, \ldots, n$ are constants.

$x^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ is the equilibrium point of (1). By the transformation $z(\cdot) = x(\cdot) - x^*$, we obtain

$$\dot{z}(t) = -Cz(t) + W_0 f(z(t)) + W_1 f(z(t - d(t))) + D\int_{t-\tau(t)}^{t} f(z(s))\mathrm{d}s, \tag{3}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T, f(z()) = [f_1(z_1()), f_2(z_2()), \ldots, f_n(z_n())]^T$ and $f_j(z_j(\cdot)) = g_j(z_j(\cdot) + x_j^*) - g_j(x_j^*), j = 1, 2, \ldots, n$. By (2), we obtain

$$k_j^- \leqslant \frac{f_j(z_j(t))}{z_j(t)} \leqslant k_j^+ \quad f_j(0) = 0, \quad j = 1, 2, \ldots, n. \tag{4}$$

Under above assumption, for positive diagonal matrix Q, we have

$$z^T(t)\bar{K}Q\bar{K}z(t) - f^T(z(t))Qf(z(t)) \geqslant 0, \tag{5}$$

where $\bar{K} = \mathrm{diag}(k_1, k_2, \ldots, k_n), k_j = max(|k_j^-|, |k_j^+|)$.

Lemma 1 [21]. *For system (3), the following inequalities are correct:*

$$0 \leqslant \int_0^{z_i(t)} (k_i^+ s - f_i(s))\mathrm{d}s \leqslant (k_i^+ z_i(t) - f_i(z_i(t)))z_i(t). \tag{6}$$

Lemma 2 [46]. *For any symmetric positive matrix $Q \in \mathscr{R}^{n\times n}$, scalars $h > 0$, then we obtain*

$$-h\int_{t-h}^{t} x^T(s)Qx(s)\mathrm{d}s \leqslant -\int_{t-h}^{t} x^T(s)\mathrm{d}sQ\int_{t-h}^{t} x(s)\mathrm{d}s. \tag{7}$$

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