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Bifurcation and exact travelling wave solutions for Gardner–KP equation

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ABSTRACT

By using the bifurcation theory of dynamical systems, this paper researches the bifurcation and exact travelling wave solutions for Gardner–KP equation. As a result, exact parametric representations of all wave solutions, including solitary wave solution, periodic wave solution, kink (anti-kink) wave solution and breaking wave solution, are given.

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1. Introduction

It is well-known the Korteweg de-Vries (KdV) equation and modified KdV (mKdV) equation respectively given by

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1}$$

and

$$u_t + au^2u_x + u_{xxx} = 0 \tag{2}$$

can be applied to model a variety of nonlinear phenomena, including ion acoustic waves in plasmas, dust acoustic solitary structures in magnetized dusty plasmas, electromagnetic waves in size-quantized films, and shallow water waves and in other applications [1]. The delicate balance between the weak nonlinearity of uu_x (or u^2u_x) and the linear dispersion of u_{xxx} gives rise to solitons that consist of single humped waves which decrease monotonically at infinity [2].

However, if the quadratic and the cubic nonlinear terms of the KdV and mKdV equation respectively are combined, then the resulting equations

$$u_t + 6uu_x \pm 6u^2u_x + u_{xxx} = 0 \tag{3}$$

are called the combined KdV–mKdV equations or Gardner equations, which describe internal solitary waves in shallow seas [3,4]. It is widely applied in various branches of physics, such as solid-state physics, plasma physics, fluid physics, quantum field theory and so on [4–6]. Depending on the sign of the cubic nonlinear term, the two models will be classified as positive and negative Gardner equation. As the KdV and mKdV equation, Gardner equation (3) are also integrable models. Many methods have been applied to solve Eq. (3), such as, Wadati's inverse scattering transform and Hirota methods [5], Coffey's series expansion method [7], Mohamads direct method [8], Lous mapping method [9] and Zhangs leading-order analysis method and direct method [10]. The application of these methods results in many kinds of exact solutions.

Kadomtsev and Petviashvili [11] extended the KdV equation to the following Kadomtsev–Petviashvili (KP) equation

$$(u_t + 6uu_x + u_{xxx})_x + u_{yy} = 0. \tag{4}$$

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They relaxed the restriction to that the waves were strictly one-dimensional, namely the x-direction of the KdV equation, and then derived the completely integrable KP equation (4). The KP equation (4) describes the evolution of quasi-one-dimensional shallow-water waves when effects of the surface tension and the viscosity are negligible.

Using the sense of Kadomtsov and Petviashivilli, Wazwaz [12] proposed the Gardner–KP (GKP) equation

$$(u_t + 6uu_x \pm 6u^2u_x + u_{xxx})_x + u_{yy} = 0, \tag{5}$$

which is completely integrable. Moreover, multiple-soliton solutions and multiple-singular soliton solutions for the GKP equation have been derived also. In addition, we had obtained several new exact travelling wave solutions via an auxiliary function method in [13]. However, the dynamical behavior of all travelling wave solutions of GKP equation has not been studied before due to some technical problems caused by the complexity of GKP equation. Compared with the auxiliary function method, or the results obtained in [13], the techniques and results in this paper have shown a complete profile of the bifurcations and phase portrait for GKP equation. Accordingly, a classification of all travelling solutions is given. Thus, this paper can be seen as a further and extended work on [13]. Interesting readers can see [12,13] for details and reference therein.

It should be noticed that GKP equation is different from the (2 + 1)-dimensional Gardner equation [14]

$$u_t + u_{xxx} + 6\beta uu_x - \frac{3}{2}\alpha^2u^2u_x + 3\sigma^2\partial_x^{-1}u_{yy} - 3\alpha\sigma u_x\partial_x^{-1}u_y = 0, \tag{6}$$

where $\sigma^2 = \pm 1$ and α, β are arbitrary constants. When $u_y = 0$, Eq. (6) reduces to the Gardner equation. For $\alpha = 0$, Eq. (6) is the KP equation.

In this paper, by using bifurcation theory from dynamical systems, all bifurcations and phase portraits of such a travelling wave system are obtained in the parametric space. Moreover, all exact travelling wave solutions corresponding to the orbits on phase portraits of Eq. (5) are derived. Meanwhile, we have shown that Eq. (5) has kink wave solutions, anti-kink wave solutions, solitary wave solutions, breaking wave solutions and infinitely many periodic wave solutions. In addition, parametric representations for all the exact travelling wave solutions are given. To the best of our knowledge, the dynamical behavior of all travelling wave solutions given by Eq. (5) has not been studied before. Therefore, this paper might be helpful to the study of GKP equation and other nonlinear differential equations.

The rest of the paper is organized as follows. In Section 2, wave transformation is constructed for reducing the GKP equations into ordinary differential equations. And then it will be further written into two planar dynamical systems which have the first integral. In Section 3, the bifurcation behaviors of travelling wave solutions on positive GKP and negative GKP equation are studied respectively. In Section 4, corresponding to all phase portraits, all possible exact parametric representations of solutions for positive GKP equation are given. In Section 5, all travelling wave solutions in the parametric space for negative GKP equation are shown.

2. Travelling wave systems

Let $u(x,y,t) = u(x+y+ct) = u(\xi)$, where c is wave speed. Substituting above travelling wave variable $\xi = x+y+ct$ into Eq. (5), we obtain

$$(cu' + 6uu' \pm 6u^2u' + u''')' + u'' = 0. \tag{7}$$

Integrating Eq. (7) once with respect to ξ and the integral constant taken as zero, we have

$$cu' + 6uu' \pm 6u^2u' + u''' + u' = 0. \tag{8}$$

Similarly, integrating Eq. (8) and neglecting the constant of integration, we obtain

$$cu + 3u^2 \pm 2u^3 + u'' + u = 0. \tag{9}$$

Positive GKP equation is equivalent to the planar integrable system

$$\frac{du}{d\xi} = \varphi, \quad \frac{d\varphi}{d\xi} = -cu - u - 3u^2 - 2u^3 \tag{10}$$

with first integral

$$H_1(u,\varphi) = \frac{1}{2}\varphi^2 + \frac{1}{2}u^4 + u^3 + \frac{1}{2}(c+1)u^2 = h_1. \tag{11}$$

Similarly, negative GKP equation is equivalent to the following system

$$\frac{du}{d\xi} = \varphi, \quad \frac{d\varphi}{d\xi} = -cu - u - 3u^2 + 2u^3 \tag{12}$$

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