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Controllability for a class of time-varying controlled switching impulsive systems with time delays ☆

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ABSTRACT

This paper investigates two types of controllability for time-varying switching impulsive control systems with time delays and the relation between the two concepts. By characterizing the solution of the system, we establish necessary and sufficient conditions of the controllability of delayed switching impulsive systems with respect to a given switching time sequence. For some special cases, linear time-varying switching impulsive systems with or without time delay, the corresponding criteria are obtained. Compared with some existing results, the results presented here are more general and less conservative. Numerical examples are provided to show the effectiveness of the proposed methods.

1. Introduction

Many real systems in physics, chemistry, biology, engineering, and information science exhibit impulsive dynamical behaviors due to abrupt changes at certain instants during the continuous dynamical processes. Hence, the study of dynamical systems with impulsive effects is of great importance and has received an increasing interest in the recent years, see [1–3] and the references therein. Nowadays, switching systems have been attracting considerable attention in the control community recently due to their theoretical development and practical applications in manufacturing, communication networks, and so on. Most efforts have focused on the controllability, and observability of switching systems [4–7]. Furthermore, since time-delay exists frequently in economic, biological and physiological systems, research on the control theory of systems with time delay has become an important topic. [8–14]. Hence, it is necessary to investigate time-varying switching impulsive systems with time delay, which has practical background in aerospace, economy, communication, etc.

Throughout the history of modern control theory and engineering, the controllability and observability play a central role since they have close connections to pole assignment, structural decomposition, quadratic optimal control and observer design. Different techniques are utilized to investigate the controllability and observability of various hybrid systems, such as fixed-point theorem for delayed systems [12,13], Lie algebraic approach for nonlinear systems[15], geometric analysis for linear systems [4–6,16], and algebraic analysis for time varying systems [12,17–20].

A commonly adopted method to deal with the controllability of delayed systems is functional analysis approach such as fixed point theory [10,11,21]. Due to the abstract conditions, the main difficulty of using the functional analysis method lies in the verification of the conditions for practical systems. In addition, the lack of analytic solution approach has limited the

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applicability of the existing theory to nonlinear systems with time delay. In [9], the criteria for controllability and observability of state delay differential equations were presented by the matrix Lambert W function. Since state delays and input delay are significantly different kinds of delays, the system solution expression and the approach employed to analyze the controllability of the two kinds of systems are also different. To the best of our knowledge, there is no result on the controllability of time-varying switched impulsive systems with delayed input. However, as an important model for dealing with complex real systems, it is necessary to consider the controllability of such systems. This motivates our current work.

In this paper, we study two types of controllability for controlled switching impulsive systems with time delay, where the impulses occur at the switching instants. A new approach based on the algebraic analysis is proposed. The explicit criteria for the controllability are presented in terms of matrix rank condition. The results are easy to check. When reduced to special cases, i.e., linear time-varying switching impulsive systems with or without time delay, the obtained criteria include some known results in the literatures. Moreover, from the numerical examples, it is interesting to find that the time delay or impulses could contribute to achieve the controllability of time-varying switched systems even if the original systems without time delay or impulses are not controllable.

The rest of this paper is organized as follows. In Section 2, the controlled switching impulsive system model is formulated and the solution of the controlled switching impulsive system is presented. In Section 3, necessary and sufficient conditions for two types of controllability of controlled switching impulsive systems with time delays are established, and the comparison with several existing results are discussed. Two numerical examples are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. Preliminaries

We consider the controlled switching impulsive systems with time delay as follows,

$$\begin{cases} \dot{x}(t) = A_{q(t)}(t)x(t) + B_{q(t)}(t)u(t) + L_{q(t)}(t)u(t-\tau), & t \in (t_{k-1}, t_k], \\ \Delta x = C_{q(t)}(t)x(t) + D_{q(t)}(t)v(t), & t = t_k, \\ x(t_0^+) = x_0, \\ u(s) = u_0(s), & s \in [t_0 - \tau, t_0], \end{cases} \tag{1}$$

where $k = 1, 2, \ldots, A_{q(t)}(t) \in \mathbf{R}^{n\times n}$, $B_{q(t)}(t) \in \mathbf{R}^{n\times b}$, $L_{q(t)}(t) \in \mathbf{R}^{n\times b}$, $C_{q(t)}(t) \in \mathbf{R}^{n\times n}$ and $D_{q(t)}(t) \in \mathbf{R}^{n\times d}$, are continuous time varying matrices, $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^b$ is a piecewise continuous time input, and $v(t) \in \mathbf{R}^d$ is a discrete time input. The left continuous time function $q(t) : \mathbf{R}^+ \to Q = \{1, 2, \ldots, r\}$ denotes the switching signal. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, where $x(t_k^+) = \lim_{h\to 0^+} x(t_k + h)$, $x(t_k) = x(t_k^-) = \lim_{h\to 0^+} x(t_k - h)$ with discontinuity points $t_0 < t_1 < t_2 < \cdots < t_M < t_{M+1} = t_f$. These discontinuity points constitute a switching time sequence, at which the impulsive behavior occur, denoted as $\Sigma = \{t_i\}_{i=1}^{M+1}$. $h_i = t_i - t_{i-1},\ h_i > \tau > 0,\ i = 1, 2 \ldots, M+1$, where τ is the time delay in the control input. $u_0(s) \in \mathbf{R}^b,\ s \in [t_0 - \tau, t_0]$ is an initial input mapping. Let $A^\top$ be the transpose of the matrix A. $\prod_{i=k}^{1} A_i$ stands for the matrix product $A_k A_{k-1} \cdots A_1$. Denote $\mathbf{N}_M = \{1, 2, \ldots, M\}$, $\mathbf{N}_{M+1} = \{1, 2, \ldots, M+1\}$.

The system model of (1) is a differential functional equation with impulsive and switching effects. Thus the dynamic behavior becomes much more complicated, which leads to the difficulty of investigating the controllability of such systems. In general, the controllability issue is considered by various fixed-point theorems based on the functional analysis [8–11,21]. While in our current work, we try to extend an algebraic approach to obtain easily verifiable conditions. For convenience, the definition of an admissible switching signal is given first.

Definition 2.1 [18]. For the given switching time sequence $\Sigma = \{t_i\}_{i=1}^{M+1}$ with $t_{M+1} = t_f$, the function $q : [t_0, t_f] \to Q$ is said to be an admissible switching signal if

(i) $q(t) \equiv q(t_0) = q(t_1)$, for $t \in [t_0, t_1]$,
(ii) $q(t) \equiv q(t_i^+) = q(t_{i+1})$, for $t \in (t_i, t_{i+1}]$, $\quad i \in \mathbf{N}_M$.

In this paper, for a given switching time sequence $\Sigma = \{t_i\}_{i=1}^{M+1}$ with $t_{M+1} = t_f$, suppose that there exists an admissible switching signal $q(t)$. Corresponding to the controlled switching impulsive system (1), for every $q(t) \in Q$, consider the differential equation

$$\dot{x}(t) = A_{q(t)}(t)x(t). \tag{2}$$

Let $X_{q(t)}(t)$ be the fundamental solution matrix of system (2). Then $X_{q(t_{i+1})}(t, s) := X_{q(t_{i+1})}(t)X^{-1}_{q(t_{i+1})}(s)$ is the transition matrix associated with the matrix $A_{q(t)}(t)$, $t, s \in (t_i, t_{i+1}],\ i = 0, 1, \ldots, M$. It is clear that $X_{q(t_{i+1})}(t, t) = I$, where I is the identity matrix of order $n, X_{q(t_{i+1})}(t, z)X_{q(t_{i+1})}(z, s) = X_{q(t_{i+1})}(t, s)$ and $X_{q(t_{i+1})}(t, s) = X^{-1}_{q(t_{i+1})}(s, t)$ for $s, t, z \in [t_0, +\infty)$. In the following, the solution of system (1) is explicitly characterized.

Lemma 2.2. *For a given switching time sequence* $\Sigma = \{t_i\}_{i=1}^{M+1}$, *and* $t \in (t_{k-1} + \tau, t_k], k = 2, \ldots, M+1$, *the solution of system* (1) *is given by*