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Delay-independent criteria for exponential admissibility of switched descriptor delayed systems ☆

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ABSTRACT

This paper deals with the uniform exponential admissibility problem for switched descriptor systems with time-varying delays. A criterion for regularity-impulsiveness-free of each subsystem is presented. A solution to switching-impulsiveness-free for switched descriptor systems is provided. A novel type of piecewise Lyapunov functionals is introduced. This type of Lyapunov functionals can efficiently overcome the switching jump of adjacent Lyapunov functionals at switching times. By applying this type of Lyapunov functionals and algebraic manipulations, the delay-independent conditions for uniform exponential admissibility is established on the minimum dwell time.

1. Introduction

As is well known, descriptor systems (known as singular, generalized or differential algebraic systems) describe interconnections of subsystems, electrical networks, robots and more generally mechanical structures, or can even be seen as approximations of singularly perturbed systems. There have been reported many works on stability analysis and control synthesis of descriptor systems, the interested reader can examine [1–6] and some references therein.

In recent years, switched systems have also received growing attention. Switched systems consist of a family of distinct active subsystems subject to a certain switching rule which chooses one of them being active during a certain time. Such systems arise, for example, when different controllers are being placed in the feedback loop with a given process, or when a given process exhibits a switching behavior caused by abrupt changes of the environment. For a discussion of various issues related to switched systems, see the survey article [7]. There are three basic problems in stability analysis and design of switched systems: (1) find conditions for stability under arbitrary switching (see, e.g., [8–11]). (2) construct an appropriate switching strategy to stabilize the system (see, for instance, [12,13]). (3) identify the limited but useful class of stabilizing switching signals (see, for example, [14–16]). To tackle these three basic problems, a considerable number of classical techniques have been proposed, such as the common Lyapunov function approach [8,9,11], the multiple Lyapunov function approach [14], the piecewise Lyapunov function approach [16], the switched Lyapunov function approach [10], and the dwell time or average dwell-time scheme [15,13].

When the singular and switching phenomena are simultaneously encountered, the switched descriptor systems are naturally arisen. The previous work for such systems mainly focuses on serval hot topics of Lyapunov stability and stabilization theory [17–22,24,23,25–27], controller synthesis [17–19,28–30,20], and reachability [31]. On the other hand, many systems, in practice, arising in disciplines, such as physics, chemistry, biology and engineering, often involve after effects or time lags.

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It has been well recognized that time delays not only degrade the performance of a control system, but also can destabilize the system [32]. Consequently, switched delayed systems have received more and more attention in recent years (see, e.g., [33,34]). At the same time, the switched descriptor delayed systems (SDDSs) starts to attract the researches' attention gradually [19,21,24,23,26,27]. Basically, the key idea is Lyapunov functional technique and average dwell time approach. Ref. [19], by the switched Lyapunov approach, focused on the robust stability and H_∞ control problems for discrete-time SDDSs under arbitrary switching, which focuses on the first basic problem. Ref. [21], for continuous-time SDDSs, presented the exponential admissibility (called exponential stability for general systems) criteria by the piecewise Lyapunov method. Ref. [24,23] extended this result to the nonlinear SDDSs. Ref. [26,27] employed the slow switching idea to stabilizable the SDDSs.

Unfortunately, there are some shortcomings for the existed results. First, all the stability criteria are subjected to the upper bounds of time delays. More precisely, these criteria are only apply the small delay cases. However, when the upper bounds of the time delays tend to infinity, it will be difficult to apply these results in the above-mentioned literature to derive the admissibility of SDDSs. Indeed, in practical applications, for many real-world control systems the upper bounds of time delays may be unknown, and some characteristics of control components may change as the process evolves. For example, the transmission delays will become large due to environmental change. If these cases occur, it is natural to expect that the increasing delays will not destroy the stability of SDDSs. To guarantee the admissibility in the face of the increasing delays, the stability conditions should be designed to be independent of the sizes of time delays. However, presenting the delay-independent conditions cannot be achieved by the existing results in [19,21,24,23,26]. Second, for switched descriptor systems, the switching impulse [22] is a unique phenomenon, and may destroy the system stability. For linear time-invariant case, Zhai and Xu [22] gave a algebraic condition to verify the existence of switching impulse. However, for the delay case, the relevant work [19,21,24,23,26] fails to deal with this problem. That is to say, when a switching impulse occurs, the stability criteria will become invalid.

The observation above inspires our research. In this paper, we are interested in the exponential admissibility of SDDSs under a class of limited switching signals. We aim to solve the two problems: (i) under what conditions, the switching impulse will not occur? (ii) derive an exponential admissibility criterion which is irrespective of the sizes of time delays, i.e., delay-independent. The layout of the paper is as follows. The notation and preliminaries are stated in Section 2. In Section 3, we first discuss the regularity-impulsiveness-free of each subsystem. Moreover, a algebraic condition will be presented to check the existence of switching impulse. Section 4 focuses on the uniform exponential admissibility. As we shall see, different from the traditional piecewise Lyapunov functional, a new class of piecewise functional is constructed to establish the minimum dwell time criterion. This criterion guaranteeing the uniform exponential admissibility of SDDSs can be applied to any bounded time delay. Two numerical examples are presented in Section 5. Finally, Section 6 concludes this paper.

2. Preliminaries

Throughout, $\mathbb{R}$ denotes the real number set. $\mathbb{R}^n$ stands for the n-dimensional real vector set and $\mathbb{R}^{n\times m}$ is the set of $n\times m$ matrices with real entries. $\mathcal{C}$ denotes the space of all real-valued continuous functions. For matrix A in $\mathbb{R}^{n\times n}, A>0(<0)$ means that A is a symmetric positive (negative) definite matrix and $A\geqslant 0(\leqslant 0)$ means that A is a symmetric positive (negative) semi-definite matrix. We use $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ to denote the smallest and largest eigenvalue of A, respectively. $\det(A)$ denotes the determination of A. $\mathbb{N}$ presents the set of all nonnegative integers and $\|\cdot\|$ denotes the Euclidean norm of vectors. For two sets $M, N, M\subseteq N$ implies that M is a subset of N. For a polynomial $p(x)$ in $x, \deg(P(x))$ denotes the highest degree.

Consider the SDDS given by

$$\begin{aligned}&E\dot{x}(t)=A_{\sigma(t)}x(t)+B_{\sigma(t)}x(t-h(t)),\\&x(\theta)=\varphi(\theta),\quad \theta\in[-h,0],\end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $\sigma(t):[0,\infty)\to\mathbb{M}$ is the switching signal, $\sigma(t)=i_k\in\mathbb{M}$ for $t\in[t_k,t_{k+1})$, $\mathbb{M}=\{1,2,\ldots,m\}, m,k\in\mathbb{N}$. Under the control of a switching signal σ, system (1) enters from the i_{k-1}th subsystem to the i_kth subsystem at the point $t=t_k, t_k$ is switching point and satisfies $t_0<t_1<\cdots<t_k<\cdots$ with $\lim_{k\to\infty}t_k=\infty$ and $t_0=0$. The time-varying delay $h(t)$ satisfies $0\leqslant h(t)\leqslant h$ and $\dot{h}(t)\leqslant d$ with given scalars h and $d<1$. The matrix $E\in\mathbb{R}^{n\times n}$ may be singular, and we assume that $rank(E)=r\leqslant n$. For each possible value $\sigma(t)=i_k$, the system matrices $A_{i_k}, B_{i_k}\in\mathbb{R}^{n\times n}$ are known constant matrices. Besides, $\varphi\in\mathcal{C}([-h,0],\mathbb{R}^n)$ is the initial function with $\|\varphi\|_h=\sup_{-h\leqslant\theta\leq 0}\|\varphi(\theta)\|$.

Assumption 2.1. In SDDS (1), there exists a positive real number τ_D such that, for any given switching signal $\sigma(t)$, $\inf_{k\in\mathbb{N}}\{t_{k+1}-t_k\}\geqslant\tau_D$.

In the literature, this is a standard assumption to rule out Zeno behavior for all types of switching [35]. How to identify or avoid Zeno phenomena is a challenging topic which is beyond the scope of this paper. In fact, τ_D satisfying the above assumption is called minimum dwell time of SDDS (1). According to the value of τ_D, we define two switching signal sets:

$$\mathcal{S}_{\min}(\tau_D)=\left\{\sigma(t)|\sigma(t)=i\in\mathbb{M},\ \inf_{k\in\mathbb{N}}\{t_{k+1}-t_k\}\geqslant\tau_D\right\}$$

and