Applied Mathematics and Computation 228 (2014) 446–453

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Properties of solution of fractional backward stochastic differential equation

Hui Zhang

School of Mathematics and Quantitative Economics, Shandong University of Finance and Economics, Jinan 250014, PR China

ARTICLE INFO

Keywords:
Backward stochastic differential equation
Fractional Brownian motion
Quasi-conditional expectation
Jensen's inequality
Comparison theorem
Comonotonic theorem

ABSTRACT

The backward stochastic differential equations (BSDEs) driven by fractional Brownian motion are studied. As an important tool, the quasi-conditional expectation is used. The general forms of Jensen's inequality of quasi-conditional expectation are proved. For the linear BSDEs, their solutions are represented by using the quasi-conditional expectation. Moreover, the comparison theorem and comonotonic theorem of the solutions of linear BSDEs are derived.

© 2013 Elsevier Inc. All rights reserved.

1. Introduction

Fractional Brownian motion with Hurst parameter $H \in (0,1)$ is a Gaussian process $B_H(t),\ t \in \mathbf{R}$, whose mean $E(B_H(t)) = 0$ and covariance $E(B_H(t)B_H(s)) = \frac{1}{2}\{|t|^{2H} + |s|^{2H} - |t-s|^{2H}\}$ for all $s, t \in \mathbf{R}$. For simplicity we assume $B_H(0) = 0$.

In this paper, we assume $H > \frac{1}{2}$. We shall consider the BSDEs driven by fractional Brownian motion which are called fractional BSDEs. Recently, stochastic calculus for fractional Brownian motion has been developed by many researchers, for detail, see [1,2]. Because linear BSDEs have played an important role in the study of stochastic control and pricing of contingent claims, we shall consider the properties of solution of the following linear BSDEs.

$$\begin{cases} dy_t = -[\alpha_t + \beta_t y_t + \gamma_t z_t]dt - z_t dB_H(t), \\ y_T = \Phi(\eta_T). \end{cases} \tag{1.1}$$

where $\alpha_t,\ \beta_t,\ \gamma_t$ are given deterministic and continuous functions, $\eta_t = \eta_0 + b_t + \int_0^t \sigma_s dB_H(s)$ with η_0 being a constant and $b_t,\ \sigma_t$ being deterministic functions.

We use the quasi-conditional expectation to study the linear BSDEs. Being applied to finance, the quasi-conditional expectation is connected with the hedging strategy. Due to its importance, we give a more detailed study of this quasi-conditional expectation. We prove the general forms of Jensen's inequality for this quasi-conditional expectation. Using the quasi-conditional expectation, we give the representation of the solution (y_t, z_t) of linear BSDEs. Applying the representation of the solution, we prove the comparison theorem and comonotonic theorem of the linear BSDEs. If you will know the comparison theorem of BSDE driven by Brownian motion, you can see Ref. [3].

This paper is organized as follows. In Section 2, we will give some notations and some basic results on fractional Brownian motion. In Section 3, the general form of Jensen's inequality for this quasi-conditional expectation will be established. In Section 4, the comparison theorem and comonotonic theorem of solution of linear BSDEs will be proved. Section 5 will summarize the conclusions of this paper.

E-mail addresses: drzhanghui@163.com, zhanghui@sdufe.edu.cn

0096-3003/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.amc.2013.11.081

2. Preliminaries and lemmas

Fix a Hurst parameter $H,\ \frac{1}{2} < H < 1$, Define

$$\phi(s,t) = H(2H-1)|s-t|^{2H-2}, \quad s,t \in [0,T]. \tag{2.1}$$

Let $f : [0,T] \to [0,T]$ be measurable. We say that $f \in L^2_\phi[0,T]$ if

$$\|f\|_T^2 := \int_0^T \int_0^T f(s)f(t)\phi(s,t)ds\,dt < \infty. \tag{2.2}$$

For any $f,g \in L^2_\phi[0,T]$, the inner product of f and g is defined by

$$\langle f,g\rangle_T := \int_0^T \int_0^T f(s)g(t)\phi(s,t)ds\,dt. \tag{2.3}$$

If we equip $L^2_\phi[0,T]$ with above inner product, then $L^2_\phi[0,T]$ is a separable Hilbert space. Let $\xi_1,\ldots,\xi_k,\ldots$ be continuous functions on $[0,T]$ such that $\langle \xi_i,\xi_j\rangle_\phi = \delta_{i,j}$. Let $\mathcal{P}_\mathcal{T}$ be the set of all elements of the form

$$F(\omega) = f\left(\int_0^T \xi_1(t)dB_H(t),\ldots,\int_0^T \xi_n(t)dB_H(t)\right),$$

where f is a polynomial of n variables.

The Malliavin derivative D_s^H of $F(\omega)$ is defined by

$$D_s^H F(\omega) = \sum_{j=1}^n \frac{\partial f}{\partial x_j}\left(\int_0^T \xi_1(t)dB_H(t),\ldots,\int_0^T \xi_n(t)dB_H(t)\right)\xi_j(s), \quad 0 \leqslant s \leqslant T. \tag{2.4}$$

Introduce also another derivative

$$\mathcal{D}_s^H F(\omega) = \int_0^T \phi(s,v)D_v^H F dv, \quad 0 \leqslant s \leqslant T. \tag{2.5}$$

The following theorems play an important role in this paper (see [4]).

Theorem 2.1. *Let $\frac{1}{2} < H < 1$ and let $f \in L^2[0,T]$ be a deterministic function. Suppose that $\|f\|_t$ is continuously differentiable as a function of $t \in [0,T]$. Denote*

$$X_t = X_0 + \int_0^t g_s ds + \int_0^t f_s dB_H(s), \quad 0 \leqslant t \leqslant T,$$

where X_0 is a constant, and g is deterministic with $\int_0^T |g_s|ds < \infty$. Let F be continuously differentiable with respect to t and twice continuously differentiable with respect to x. Then

$$F(t,X_t) = F(0,X_0) + \int_0^t \frac{\partial F}{\partial s}(s,X_s)ds + \int_0^t \frac{\partial F}{\partial x}(s,X_s)dX_s + \frac{1}{2}\int_0^t \frac{\partial^2 F}{\partial x^2}(s,X_s)\left[\frac{d}{ds}\|f\|_s^2\right]ds, \quad 0 \leqslant t \leqslant T.$$

Theorem 2.2. *Let $f_i(s),\ g_i(s),\ 0 \leqslant s \leqslant T$ be real valued stochastic processes such that*

$$E\left[\int_0^T |f_i(s)|^2 ds + \int_0^T |g_i(s)|^2 ds\right] < \infty, \quad i = 1,2.$$

Assume also that $\mathcal{D}_t^H f_2(s)$ and $\mathcal{D}_t^H g_2(s)$ are continuously differentiable with respect to $s,t \in [0,T]$ for almost all $\omega \in \Omega$. Suppose that $E[\int_0^T \int_0^T |\mathcal{D}_t^H f_2(s)|^2 ds\,dt] < \infty$, and $E[\int_0^T \int_0^T |\mathcal{D}_t^H g_2(s)|^2 dsdt] < \infty$. Denote

$$F(t) = \int_0^t f_1(s)ds + \int_0^t f_2(s)dB_H(s), \quad 0 \leqslant t \leqslant T.$$

$$G(t) = \int_0^t g_1(s)ds + \int_0^t g_2(s)dB_H(s), \quad 0 \leqslant t \leqslant T.$$

Then

$$F(t)G(t) = \int_0^t F(s)g_1(s)ds + \int_0^t F(s)g_2(s)dB_H(s) + \int_0^t G(s)f_1(s)ds + \int_0^t G(s)f_2(s)dB_H(s) + \int_0^t \mathcal{D}_s^H F(s)g_2(s)ds + \int_0^t \mathcal{D}_s^H G(s)f_2(s)ds, \quad 0 \leqslant t \leqslant T.$$

Download English Version:

https://daneshyari.com/en/article/4628354

Download Persian Version:

https://daneshyari.com/article/4628354

Daneshyari.com