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Spiral slits map and its inverse of bounded multiply connected regions

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ABSTRACT

This paper presents a boundary integral equation method for computing numerical conformal mapping of bounded multiply connected region onto a spiral slits region. The method is an extension of the author's method for computing the circular slits map of bounded multiply connected regions (see A.W.K. Sangawi et al. (2012) [16]). Several numerical examples are given to prove the effectiveness of the proposed methods.

1. Introduction

A boundary integral equation method for mapping a bounded multiply connected region onto a circular slits region has been presented in [16]. The classical books [14,1] described the five classical canonical slit regions. The spiral slits region includes as special cases the canonical slits regions considered in Koebe [6], DeLillo et al. [4] and Nasser [11]. This region has been studied for bounded and unbounded multiply connected regions in [11] by reformulating the conformal mapping as a Riemann–Hilbert problem. The main idea of this paper is to modify a previously known construction of conformal mapping to a circular slit region, to produce instead a region whose jth boundary component is a logarithmic spiral

$$\operatorname{Im} e^{-i\theta_j} \log w = \text{constant},$$

where the oblique angles θ_j are prescribed in advance. In this paper, we extend the result of [16] by presenting a boundary integral equation method for numerical conformal mappings from a bounded multiply connected region onto the spiral slits region. The proposed method is based on linear boundary integral equation method which is constructed from a boundary relationship satisfied by an analytic function on a bounded multiply connected region.

The plan of the paper is as follows: Section 2 presents some notations and auxiliary materials. Section 3 presents a method to calculate the piecewise real functions h_j and υ_j. Section 4 presents a derivation of integral equation related to S'. In Section 5, we give some examples to illustrate our boundary integral equation method. Finally, Section 6 presents a short conclusion.

2. Notations and auxiliary material

A bounded multiply connected region Ω of connectivity $M+1$. The boundary Γ consists of $M+1$ smooth Jordan curves $\Gamma_j,\ j = 0, 1, \ldots, M$ as shown in Fig. 1.

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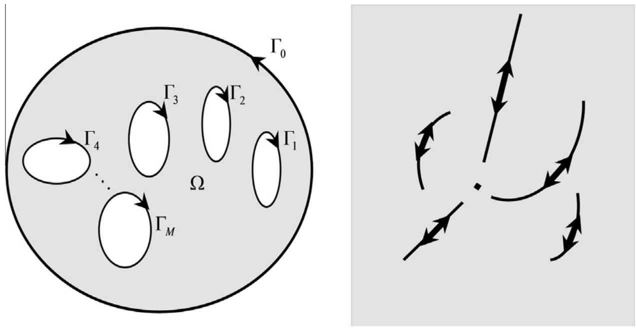

Fig. 1. Mapping of the bounded multiply connected region onto a spiral slits region.

The curve Γ_j is parametrized by a 2π-periodic twice continuously differentiable complex function $z_j(t)$ with non-vanishing first derivative

$$z_j'(t) = dz_j(t)/dt \neq 0, \quad t \in J_j = [0, 2\pi], \quad j = 0, 1, \ldots, M.$$

The total parameter domain J is the disjoint union of $M+1$ intervals $J_0, \ldots, J_M$. We define a parametrization $z(t)$ of the whole boundary Γ on J by

$$z(t) = z_j(t), \quad t \in J_j, \quad j = 0, 1, \ldots, M. \tag{1}$$

The unknown functions $S(t)$ and $R(t)$ (which will be defined in Section 4) will be given for $t \in J$ by

$$S(t) = S_j(t) \text{ and } R(t) = R_j(t), \quad t \in J_j,\ j = 0, 1, \ldots, M.$$

Suppose that $c(z), Q(z)$ and $H(z)$ are complex-valued functions defined on Γ such that $c(z) \neq 0, H(z) \neq 0$, $Q(z) \neq 0$ and $\overline{H(z)}/(T(z)Q(z))$ satisfies the Hölder condition on Γ. Then the interior relationship is defined as follows:

A complex-valued function $P(z)$ is said to satisfy the interior relationship if P(z) is analytic in Ω and satisfies the non-homogeneous boundary relationship

$$P(z) = c(z)\frac{\overline{T(z)Q(z)}}{\overline{G(z)}}\overline{P(z)} + \overline{H(z)}, \quad z \in \Gamma, \tag{2}$$

where $G(z)$ analytic in Ω, Hölder continuous on Γ, and $G(z) \neq 0$ on Γ. The boundary relationship (2) also has the following equivalent form:

$$G(z) = \overline{c(z)}T(z)Q(z)\frac{P(z)^2}{|P(z)|^2} + \frac{G(z)H(z)}{\overline{P(z)}}, \quad z \in \Gamma. \tag{3}$$

The following theorem gives an integral equation for an analytic function satisfying the interior non-homogeneous boundary relationship (2) or (3), [16].

Theorem 1. *Let U and V be any complex-valued functions that are defined on Γ. If the function $P(z)$ satisfies the interior non-homogeneous boundary relationship (2) or (3), then*

$$\frac{1}{2}\left[V(z) + \frac{U(z)}{\overline{T(z)Q(z)}}\right]P(z) + \mathrm{PV}\int_\Gamma K(z,w)P(w)|dw| + c(z)U(z)\left[\sum_{a_j \in \Omega} \operatorname*{Res}_{w=a_j} \frac{P(w)}{(w-z)G(w)}\right]^{\mathrm{conj}} = -U(z)\overline{L_R^-(z)}, \quad z \in \Gamma, \tag{4}$$

where

$$K(z,w) = \frac{1}{2\pi \mathrm{i}}\left[\frac{c(z)U(z)}{\overline{c(w)(\overline{w} - \overline{z})Q(w)}} - \frac{V(z)T(w)}{w - z}\right], \tag{5}$$

$$L_R^-(z) = \frac{-1}{2}\frac{H(z)}{Q(z)T(z)} + \mathrm{PV}\frac{1}{2\pi \mathrm{i}}\int_\Gamma \frac{\overline{c(z)}H(w)}{\overline{c(w)}(w-z)Q(w)T(w)}dw. \tag{6}$$

The symbol "conj" in the superscript denotes complex conjugate, while the minus sign in the superscript denotes limit from the exterior.