Applied Mathematics and Computation 228 (2014) 606–614

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Global output feedback stabilization for a class of nonlinear time-varying delay systems

Jun-yong Zhai

Key Laboratory of Measurement and Control of CSE, Ministry of Education, School of Automation, Southeast University, Nanjing, Jiangsu 210096, China

ARTICLE INFO

Keywords:
Time-varying delay
Homogeneous domination
Output feedback
Nonlinear systems

ABSTRACT

This paper investigates the problem of global stabilization by output feedback for a class of nonlinear systems with time-varying delays. The uncertain nonlinearities are assumed to satisfy a polynomial growth condition. We first design an output feedback stabilizer to globally stabilize the nominal system without the perturbing nonlinearities. Then based on the homogeneous domination approach and the appropriate Lyapunov–Krasovskii functional, a scaling gain is introduced into the proposed output feedback stabilizer to render the closed-loop system globally asymptotically stable. Two simulation examples are given to illustrate the effectiveness of the proposed design scheme.

1. Introduction

The problem of output feedback stabilization of nonlinear systems has attracted a great deal of attention from the nonlinear control community and a series of research results have been achieved [1–9] and the references therein. However, as shown in [1], usually the separation principle does not hold for general nonlinear systems. Hence, certain conditions are required for the existing global results. Among the different kinds of assumptions, one common condition is that the unmeasurable states cannot be associated with the uncertainties. To deal with this case, a feedback domination design method has been proposed in [10] under a linear growth condition. Later [2] (recently extended in [11,12]) has solved the problem of global output feedback stabilization under a higher-order growth condition by employing the homogeneous domination approach. In [3] the problem of global output feedback stabilization was considered for nonlinear systems with unknown output functions.

Time-delay often appears in many practical control systems, such as communication systems, engineering systems, process control systems, high-speed networks, etc. The delay is a source of the instability and oscillatory response, and thus the stability analysis and control design of the time-delay systems are of great importance for both theoretical and practical reasons [13–17]. In the past two decades, many nice results have been obtained for the analysis and synthesis of both linear time-delay systems and nonlinear time-delay systems. The stability criteria for time-delay systems can be divided into two cases: the delay-independent and delay-dependent ones. Since the delay-independent criterion can lead to more conservatism for small-size delay systems, considerable attention has been devoted to the delay-dependent one. In addition, many adaptive control schemes have been proposed for nonlinear time-delay systems, see, e.g. [18–23] and the references therein. A tracking control problem has been considered for a class of nonlinear time-delay systems with nonsymmetric dead-zone input in [18]. The time-delay uncertainties are bounded by nonlinear function with unknown coefficients. A smooth adaptive controller is constructed such that the solution of the resulting closed-loop system exponentially converges to an adjustable region. An adaptive stabilizer has been proposed for feedforward nonlinear systems with time delays in [19]. The stabilizer

E-mail address: jyzhai@seu.edu.cn

0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2013.12.019

takes a nested saturation feedback, and a set of switching logics are designed to tune online the saturation levels in a switching manner. Adaptive controllers based on the backstepping technique without modification are shown to be robust with respect to time delay in system input and unmodeled dynamics in [20]. The adaptive stabilization problem for nonlinear time-delay systems has been investigated in [21]. A universal-type adaptive output feedback controller has been proposed for a class of uncertain nonlinear systems with unknown time delays in [22]. Based on the Lyapunov–Razumikhin theorem and Lyapunov–Krasovskii theorem, the delay-independent feedback controllers have been explicitly constructed such that the closed-loop systems are globally asymptotically stable in [23].

In spite of these developments, to the authors' knowledge, the theory of stability analysis for uncertain nonlinear systems with time-varying delays has not yet been fully developed. In this paper, we will consider the problem of global output feedback stabilization for a class of uncertain nonlinear time-varying delay systems described by

$$\begin{aligned}
\dot{z}_i(t) &= z_{i+1}(t) + \phi_i(t, z(t), z(t - d_i(t)), v(t)), \quad i = 1, \ldots, n-1, \\
\dot{z}_n(t) &= v(t) + \phi_n(t, z(t), z(t - d_n(t)), v(t)), \\
y(t) &= \theta z_1(t),
\end{aligned} \tag{1}$$

where $z(t) = (z_1(t), \ldots, z_n(t))^T \in \mathbb{R}^n$, $v(t) \in \mathbb{R}, y(t) \in \mathbb{R}$ are the system states, control input and output, respectively. $\phi_i : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}, i = 1, \ldots, n$, are continuous nonlinear functions. θ is an unknown constant satisfying $0 < \underline{\theta} \leqslant \theta \leqslant \overline{\theta}$. The time-varying delays $d_i(t), i = 1, \ldots, n$, satisfy $0 \leqslant d_i(t) \leqslant h_i$ and $\dot{d}_i(t) \leqslant \gamma_i < 1$ for constants h_i and γ_i, which are Borel measurable functions and h is a known constant, which denotes the upper bound of all time-delays, namely, $0 < d_i(t) \leqslant h$ for $i = 1, \ldots, n$. For simplicity, the time variable t in the delay-free states will be omitted in the rest of this paper.

Notation. $\mathbb{R}_+$ denotes the set of all nonnegative real numbers and $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $|X|$ denotes the Euclidean norm of a vector X. $\mathcal{C}([-h, 0]; \mathbb{R}^n)$ denotes the space of continuous $\mathbb{R}^n$-value functions on $[-h, 0]$ endowed with the norm $\|\cdot\|$ defined by $\|f\| = \sup_{x \in [-h,0]} |f(x)|$ for $f \in \mathcal{C}([-h, 0]; \mathbb{R}^n)$; $\mathcal{C}^b_{\mathcal{F}_0}([-h, 0]; \mathbb{R}^n)$ denotes the family of all $\mathcal{F}_0$-measurable bounded $\mathcal{C}([-h, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\Theta) : -h \leqslant \Theta \leqslant 0\}$. $\mathcal{K}$ denotes the set of all functions: $\mathbb{R}_+ \to \mathbb{R}_+$, that are continuous, strictly increasing and vanishing at zero; $\mathcal{K}_\infty$ denotes the set of all functions that are of class $\mathcal{K}$ and unbounded; $\mathcal{KL}$ denotes the set of all functions $\beta(s, t) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, that are of $\mathcal{K}$ for each fixed t and that decrease to zero as $t \to \infty$ for each fixed s.

2. Preliminaries

This section contains a definition and several useful lemmas which play important roles in this paper.

Definition 2.1 [24]. For real numbers $r_i > 0, i = 1, \ldots, n$ and fixed coordinates $(x_1, \ldots, x_n) \in \mathbb{R}^n$:

- the dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x) = (\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n)$, for all $\varepsilon > 0$, with r_i being called as the weights of the coordinates. For simplicity, we define dilation weight $\Delta = (r_1, \ldots, r_n)$.
- a function $V \in \mathcal{C}(\mathbb{R}^n, \mathbb{R})$ is said to be homogeneous of degree τ if there is a real number $\tau \geqslant 0$ such that

  $$\forall x \in \mathbb{R}^n \setminus \{0\}, \quad V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1, \ldots, x_n).$$

- a vector field $f \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^n)$ is said to be homogeneous of degree τ if there is a real number $\tau \geqslant -\min\{r_1, \ldots, r_n\}$ such that for $i = 1, \ldots, n$

  $$\forall x \in \mathbb{R}^n \setminus \{0\}, \quad f_i(\Delta_\varepsilon(x)) = \varepsilon^{\tau + r_i} f_i(x_1, \ldots, x_n).$$

- a homogeneous p-norm is defined as $\|x\|_{\Delta,p} = \left(\sum_{i=1}^n |x_i|^{p/r_i}\right)^{1/p}, \forall x \in \mathbb{R}^n$, for a constant $p \geq 1$. For simplicity, we choose $p = 2$ and write $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$.

Lemma 2.1 [25]. *Suppose $V : \mathbb{R}^n \to \mathbb{R}$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then the following hold:*

(i) $\partial V / \partial x_i$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i;
(ii) There is a constant $\bar{c}$ such that

$$V(x) \leqslant \bar{c} \|x\|^\tau_\Delta.$$

Moreover, if $V(x)$ is positive definite, then there exists a constant $\underline{c} > 0$, such that

$$V(x) \geqslant \underline{c} \|x\|^\tau_\Delta.$$