Applied Mathematics and Computation 226 (2014) 10–19

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Delay-dependent stability analysis for a class of dynamical systems with leakage delay and nonlinear perturbations

Xiaodi Li [a,*], Xilin Fu [a], R. Rakkiyappan [b]

[a] School of Mathematical Sciences, Shandong Normal University, Ji'nan 250014, Shandong, PR China
[b] Department of Mathematics, Bharathiar University, Coimbatore 641 046, India

ARTICLE INFO

Keywords:
Dynamical systems
Leakage delay
Global asymptotic stability
Delay-dependent
Nonlinear perturbations

ABSTRACT

This paper studies the stability problem for a class of dynamical systems with leakage delay and nonlinear perturbations based on linear matrix inequality (LMI) approach. Some sufficient conditions which are dependent on the leakage delay are derived to ensure the global asymptotic stability by using Lyapunov–Krasovskii functional method and free weighting matrix technique. Two examples and their simulations are given to show the effectiveness and advantage of the present results.

1. Introduction

As we know, time delay is commonly encountered in various physical and engineering systems such as the turbojet engine, microware oscillator, nuclear reactor, rolling mill, chemical process and hydraulic systems [1–3]. Moreover, the existence of time delay in a real system may induce instability, oscillation, and bad dynamic performance [4]. Hence, it is significant and necessary to consider the delay effects on stability of dynamical systems. Recently, many interesting results on stability of dynamical systems with time delays have been reported in the literature [5–12]. In general, they can be classified into two categories: delay-dependent and delay-independent results. Since delay-dependent results make use of information on the length of delays, they are usually less conservative than delay-independent ones. With the rapid development of networks technology, various techniques have been developed and explored to derive the delay-dependent stability criteria for delayed systems, especially for uncertain delayed systems, see [13–24]. For instance, Han [15] obtained some delay-dependent stability criteria for uncertain linear systems with time-varying delay by employing a descriptor model transformation and a decomposition technique of the delay term matrix. By employing Lyapunov–Krasovskii functional methods and free weighting matrices techniques, Wu et al. [16] presented some delay-dependent stability criteria for a class of time-varying delay systems, which improves the existing results in [17–19]. In [20], Yue et al. proposed a novel piecewise analysis method to derive some delay-dependent stability criteria for linear delayed continuous/discrete systems with uncertainty. The obtained results are less conservative than those given in [21–23].

Recently, a special type of time delay, namely, leakage delay (or forgetting delay), is identified and investigated due to its existence in many real systems such as neural networks, population dynamics and some fuzzy systems [25–27]. Leakage delay is a time delay that exists in the negative feedback terms of the system which are known as forgetting or leakage terms. It has been shown that such kind of time delay has a tendency to destabilize a system [28,30]. Recall the past several years, however, there is little work done on dynamics of systems with leakage delay [28–35]. Recently, the existence, uniqueness and global asymptotic stability analysis of impulsive neural networks with leakage delay has been investigated in [30] by using contraction mapping theorem, topological degree theory, Lyapunov–Kravsovskii functional and some analysis

* Corresponding author.
E-mail address: sodymath@163.com (X. Li).

0096-3003/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.amc.2013.10.004

techniques. In [31], some delay-independent and delay-dependent sufficient conditions for stability of nonlinear systems with leakage delay were presented by using contraction mapping theorem, Lyapunov–Kravsovskii functional and LMIs. Since free weighting matrices method proves to be a powerful tool to improve the performance of delay-dependent stability criteria for delay dynamical systems, it is interesting to develop this method to dynamical systems with leakage delay. In addition, it is well known that nonlinearities, as time delays, exists widely in practice such as telecommunication, neural networks, economic systems and many chemical processes and may cause oscillation, instability and poor performance of real systems, which have driven many researchers to study the problem of nonlinear perturbed systems with time delays during recent years [10,13–15,32–34].

With the above motivation, the main aim of this paper is to provide a further contribution to the LMIs of delay-dependent stability criteria for a class of dynamical systems with leakage delay and nonlinear perturbations. We introduce a new Lyapunov–Krasovskii functional by taking the information of integral terms with leakage delay and derivative of variables into account and moreover, the leakage delay occurs in both of single and double integral terms in order to derive the desirable results. Some novel global asymptotic stability criteria which are dependent on the leakage delay are derived under the help of some free weighting matrices and Leibniz–Newton formula. The proposed criteria are formulated in terms of a set of LMIs which can be easily solved using efficient convex optimization algorithms. Moreover, our development results improve and generalize the results in [31]. Two examples and their simulations are given to show the effectiveness and advantage of the present results.

2. Preliminaries

Notations. Let $\mathbb{R}$ ($\mathbb{R}^+$) denote the set of (positive) real numbers, $\mathbb{Z}_+$ denote the set of positive integers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional and $n\times m$-dimensional real spaces equipped with the Euclidean norm $\|\bullet\|$, respectively. $\mathcal{A}>0$ or $\mathcal{A}<0$ denotes that the matrix $\mathcal{A}$ is a symmetric and positive definite or negative definite matrix. The notation $\mathcal{A}^T$ and $\mathcal{A}^{-1}$ mean the transpose of $\mathcal{A}$ and the inverse of a square matrix. If $\mathcal{A},\mathcal{B}$ are symmetric matrices, $\mathcal{A}>\mathcal{B}$ means that $\mathcal{A}-\mathcal{B}$ is positive definite matrix. $\lambda_{\max}(\mathcal{A})$ and $\lambda_{\min}(\mathcal{A})$ denote the maximum eigenvalue and the minimum eigenvalue of matrix $\mathcal{A}$, respectively. I denotes the identity matrix with appropriate dimensions. For any interval $J\subseteq\mathbb{R}$, set $S\subseteq\mathbb{R}^k(1\leqslant k\leqslant n), C(J,S)=\{\phi:J\to\ S$ is continuous$\}$ and $C^1(J,S=\{\varphi:J\to\ S$ is continuously differentiable$\}$. In particular, let $\mathbb{C}_r^1\doteq C^1([-r,0],\mathbb{R}^n)$ for some constant $r>0$. For $\varphi\in\mathbb{C}_r^1$, the norm is defined as $\|\varphi\|_r=\max\{\sup_{-r\leqslant s\leqslant 0}\|\varphi(s)\|,\sup_{-r\leqslant s\leqslant 0}\|\dot{\varphi}(s)\|\}$. In addition, the notation $\star$ always denotes the symmetric block in one symmetric matrix.

Consider the following dynamical system:

$$\begin{cases}\dot{x}(t) &= -Ax(t-\sigma)+Bx(t-\tau)+f(t,x(t-\sigma),x(t-\tau)),\quad t>0,\\ x(t) &= \phi(t),\quad t\in[-\rho,0],\end{cases}\tag{1}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in\mathbb{R}^n$ is the state vector; $\sigma\geqslant 0$ is the leakage delay and $\tau\geqslant 0$ is the transmission delay; $\phi\in\mathbb{C}_\rho^1,\rho\doteq\max\{\sigma,\tau\};A\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{n\times n}$ are two constant matrices; $f(t,x(t-\sigma),x(t-\tau))$ represents the nonlinear term of system (1) which satisfies $f(t,0,0)=0$ and

$$\|f(t,x(t-\sigma),x(t-\tau))\|\leqslant\alpha\|E_\alpha x(t-\sigma)\|+\beta\|E_\beta x(t-\tau)\|,\tag{2}$$

where α and β are two nonnegative real constants, E_α and E_β are two known real matrices.

Rewrite system (1) in the following equivalent descriptor system

$$\begin{cases}\frac{d}{dt}\left[x(t)-A\int_{t-\sigma}^t x(s)ds\right]=-Ax(t)+Bx(t-\tau)+f(t,x(t-\sigma),x(t-\tau)),\quad t>0,\\ x(t)=\phi(t),\quad t\in[-\rho,0].\end{cases}\tag{3}$$

In this paper, the following Lemma will be used for deriving our main result.

Lemma 2.1 [36]. Given any real matrix $M>0$ of appropriate dimension and a vector function $\omega(\cdot):[a,b]\to\mathbb{R}^n$, such that the integrations concerned are well defined, then

$$\left[\int_a^b\omega(s)ds\right]^T M\left[\int_a^b\omega(s)ds\right]\leqslant(b-a)\int_a^b\omega^T(s)M\omega(s)ds.$$

3. Main results

Theorem 3.1. *Given scalars $\sigma\geqslant 0$ and $\tau\geqslant 0$, system (1) is globally asymptotically stable if there exist a constant $\varepsilon>0$, four positive definite matrices $P>0,\ Q_i>0,\ i=1,2,3,$ five real matrices $R_i,\ i=1,\ldots,5,$ two semi-positive-definite matrices*

$$X=\begin{bmatrix}X_{11}&X_{12}&X_{13}\\ \star&X_{22}&X_{23}\\ \star&\star&X_{33}\end{bmatrix}\geqslant 0,\quad Y=\begin{bmatrix}Y_{11}&Y_{12}&Y_{13}\\ \star&Y_{22}&Y_{23}\\ \star&\star&Y_{33}\end{bmatrix}\geqslant 0$$