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A two parameter variance estimator using auxiliary information

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ABSTRACT

We propose a two parameter ratio-product-ratio estimator for estimating a finite population variance based on simple random sampling without replacement. The bias and mean square error of the proposed estimator are obtained to the first degree of approximation. We have derived the conditions for the parameters under which the proposed estimator has smaller mean square error than the sample variance, ratio and product estimators. We carry out an application showing that the proposed estimator outperforms the traditional estimators using different data sets.

1. Introduction

Auxiliary information is in use since the use of sampling itself and it is used to enhance the efficiency of the estimators for estimating the population parameters.

Let $(x_i, y_i), i = 1, 2, \ldots\ldots\ldots, n$ be the n pair of sample observations for the auxiliary and study variables, respectively, drawn from the population of size N using Simple Random Sampling Without Replacement. Let $\overline{X}$ and $\overline{Y}$ be the population means of the auxiliary and study variables, respectively and $\bar{x}$ and $\bar{y}$ be the respective sample means. Ratio estimators are used when the line of regression of y on x passes through origin and the variables X and Y are positively correlated to each other, while product estimators are used when X and Y are negatively correlated to each other.

The sample variance estimator of the population variance is defined as

$$t_0 = s_y^2 \tag{1.1}$$

which is an unbiased estimator of the population variance $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2$ and its variance is

$$V(t_0) = \gamma S_y^4(\lambda_{40} - 1), \tag{1.2}$$

where $\gamma = \frac{1}{n}$.

Isaki [9] proposed the ratio type estimator for the population variance of the study variable as

$$t_R = s_y^2\left(\frac{S_x^2}{s_x^2}\right), \tag{1.3}$$

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where

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2, \quad s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2, \quad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$

,

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N}X_i, \quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N}Y_i, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i.$$

The Bias and Mean Square Error (MSE) of the estimator in (1.3), up to the first order of approximation are given, respectively, as

$$B(t_R) = \gamma S_y^2[(\lambda_{40} - 1) - (\lambda_{22} - 1)], \tag{1.4}$$

$$MSE(t_R) = \gamma S_y^4[(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)], \tag{1.5}$$

where $\lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$ and $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^r(X_i - \bar{X})^s$,

The product type estimator for the population variance of the study variable based on Robson [18] product estimator is given as

$$t_P = s_y^2\left(\frac{s_x^2}{S_x^2}\right). \tag{1.6}$$

The Bias and Mean Square Error (MSE) of the estimator in (1.6), up to the first order of approximation, are given, respectively, as

$$B(t_P) = \gamma S_y^2[(\lambda_{40} - 1) + (\lambda_{22} - 1)], \tag{1.7}$$

$$MSE(t_P) = \gamma S_y^4[(\lambda_{40} - 1) + (\lambda_{04} - 1) + 2(\lambda_{22} - 1)]. \tag{1.8}$$

Many more authors, including Evans [6], Liu [15], Royall and Eberhardt [19], Das and Tripathi [4], Srivastava and Jhajj [24], Wu [29], Singh et al. [23], Mukhopadhyaya [16], Prasad and Singh [17], Garcia and Cebrain [7], Upadhyaya and Singh [27], Upadhyaya and Singh [28], Singh and Singh [21], Kadilar and Cingi [10], Kadilar and Cingi [11], Kadilar and Cingi [12], Kadilar and Cingi [13], Kadilar and Cingi [14], Arcos et.al. [2], Dubey and Sharma [5], Gupta and Shabbir [8], Al-Hadhrami [1], Singh et al. [20], Singh et al. [22], Subramani and Kumarapandiyan [25], Subramani and Kumarapandiyan [26], Chami et al. [3] etc. have contributed to variance estimation.

2. Material and methods

After examining the related estimators in the studies mentioned in Section 1, for estimating the population variance of the main variable under study and motivated by Chami et al. [3] estimator of population mean given by,$\bar{y}_{\alpha,\beta} = \alpha\left[\frac{(1-\beta)\bar{x}+\beta\bar{X}}{\beta\bar{x}+(1-\beta)\bar{X}}\right]\bar{y} + (1-\alpha)\left[\frac{\beta\bar{x}+(1-\beta)\bar{X}}{(1-\beta)\bar{x}+\beta\bar{X}}\right]\bar{y}$,
where α and β are real constants, we propose the following two parameter ratio-product-ratio estimator

$$t_{\alpha,\beta} = \alpha\left[\frac{(1-\beta)s_x^2 + \beta S_x^2}{\beta s_x^2 + (1-\beta)S_x^2}\right]s_y^2 + (1-\alpha)\left[\frac{\beta s_x^2 + (1-\beta)S_x^2}{(1-\beta)s_x^2 + \beta S_x^2}\right]s_y^2, \tag{2.1}$$

Our goal in this article is to derive the values for these constants α and β such that the bias and/or the mean square error (MSE) of $t_{\alpha,\beta}$ is minimal.

Note that $t_{\alpha,\beta} = t_{(1-\alpha),(1-\beta)}$ i.e., the estimator $t_{\alpha,\beta}$ is invariant under a point reflection through $(\alpha,\beta) = \left(\frac{1}{2},\frac{1}{2}\right)$. In the point of symmetry $(\alpha,\beta) = \left(\frac{1}{2},\frac{1}{2}\right)$, the proposed estimator reduces to the sample variance, i.e., we have $t_{\frac{1}{2},\frac{1}{2}} = s_y^2$. In fact, on the whole line $\beta = \frac{1}{2}$, the proposed estimator reduces to the sample variance estimator, i.e., $t_{\alpha,\frac{1}{2}} = s_y^2$. Similarly, we get $t_{1,0} = t_{0,1} = s_y^2\left(\frac{s_x^2}{S_x^2}\right) = t_P$ (product estimator), and $t_{0,0} = t_{1,1} = s_y^2\left(\frac{S_x^2}{s_x^2}\right) = t_R$ (ratio estimator).