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Noisy chaos in intraday financial data: Evidence from the American index

Ahmed BenSaïda *

LaREMFiQ, IHEC – University of Sousse, Tunisia

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ABSTRACT

The presence of chaos in financial markets was inconclusive due mainly to test misspecification and data type. Although noisy chaos was investigated in recent studies, it was only explored in daily returns, which does not necessary mean that continuous intra-daily data will exhibit the same dynamics. High level noisy chaos is tested in the Standard & Poor's 500 index returns over 4 different frequencies: weekly, daily, 30-min and 5-min basis; the dynamics in all frequencies are non-chaotic.

1. Introduction

The dynamics of financial returns are usually considered as stochastic, and governed by some probability distribution function. However, although stochastic models are consistent in in-sample modeling, they have poor forecasting power when it comes to financial data. Recently, some studies suggest that these dynamics are actually chaotic and not stochastic [1–4], and others have found that these dynamics are stochastic [5,6]. Nevertheless, chaotic behavior were only investigated in daily returns; depending on the data type and the used test, results in the literature were inconclusive.

Chaos is a field of study which has been observed in physics since 1960s. The predominant linear techniques at that time were unable to explain some specific phenomena such as the movement of a driven pendulum where it may behave erratically and show irregular sequences of left and right turns. Recently, in economic and financial sciences, the debate still stands trying to find out whether financial markets are primary generated by stochastic or chaotic dynamics. These two systems look almost the same and even the powerful BDS test [7] for IID cannot separate them. Consequently, two main streams describing the stock behavior have seen the light, and until then the distinction between them depends on the theoretical context. Though, recent chaos tests are efficient only for clean measured data such as observations in physics, and are not valid for financial data which are subject to measurement noise. It was until a recent study where a practical test for noisy chaos, which can be applied to financial scalar time series, has been developed [6]. Yet, investigating chaos in financial markets, were only applied to daily frequency; although one particularity in financial data is that we can collect them for different frequencies, even intra-daily. Hence, rejecting or accepting chaos for daily returns, does not necessary imply that continuous intra-daily data would behave the same way. The remaining of this paper is organized as follow: Section 2 discusses the concepts and methodology, Section 3 investigates the dynamics of the American S&P 500 index returns for different frequencies, and Section 4 deliberates some concluding remarks.

* Address: B.P. 40, Route de la ceinture, Sahloul 3, 4054 Sousse, Tunisia.
E-mail address: ahmedbensaida@yahoo.com

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2. Concepts and methodology

In a scientific context, chaos refers to an apparent lack of order in a system that nevertheless obeys particular laws or rules. The two main components of chaos theory are the ideas that systems – no matter how complex they may be – rely upon an underlying order, and that very simple or small systems and events can cause very complex behaviors or events, also known as *sensitive dependence on initial conditions*.

There are many studies of the mathematical aspects of chaos and dynamical systems, including Eckmann and Ruelle [8] and Devaney [9]. Numerical implementations are discussed in Parker and Chua [10]. Since then, Chaos has attracted attention of the statistical community, which includes Smith [11], and Casdagly [12]. Moreover, recognizing and quantifying chaos in time series was the subject of many studies. In fact, several approaches have been proposed including estimating fractal dimensions [11], nonlinear forecasting [12], estimating entropy [8], and estimating the dominant Lyapunov exponent [13,14].

Chaotic behavior in financial markets was first tested in the literature via the BDS test [15–17]. However, this test is omnipotent in detecting nonlinearity whether it is due to stochastic or chaotic dynamics; since then the search for an alternative test for chaos applicable to financial time series is still going on, including the close returns test [5], and the Lyapunov exponent [1,2,4,18–20].

Still, a comparison between the close returns test, the BDS test and the Lyapunov exponent in its current form did not provide any conclusive results [21]. When applied to financial time series, the Lyapunov exponent is positive although the series is known to be non-chaotic. This is mainly due to the inadequacy of the use of the Lyapunov exponent on noisy data such as financial time series [6].

2.1. The Lyapunov exponent

Consider one point in a state space X_0 that generates a reference orbit using some equation or system of equations. Adding a perturbation Δx_0 in the initial condition X_0 would generate a perturbed orbit. The separation between the two orbits after time t will be a function of time $\Delta x(X_0, t)$. The Lyapunov exponent λ measures the average exponential divergence (positive exponent) or convergence (negative exponent) rate between nearby trajectories within a time horizon that differ in initial conditions only by an infinitesimally small amount, Eq. (1). Chaotic systems exhibit a positive λ, and stable non-chaotic systems exhibit a strictly negative λ.

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \frac{|\Delta x(X_0, t)|}{|\Delta x_0|} \tag{1}$$

Estimating the Lyapunov exponent for a scalar time series is not evident, since we cannot alter the initial conditions. Wolf et al. [13] have developed a direct approach based on arbitrarily defining an Ordinary Differential Equation of the chaotic system that the observed data may follow, and test whether the data process coincides with the predefined system or not. This approach has limited applications since it bounds the chaotic behavior to the tested ones. Moreover, it cannot accept measurement errors or noise. Another method based on the Jacobian approach, proposed by Eckmann and Ruelle [8], can efficiently handle noisy systems [6].

Given a scalar time series $\{x_t\}_{t=1}^{T}$, a noisy chaotic system can be written as:

$$x_t = f(x_{t-L}, x_{t-2L}, \ldots, x_{t-mL}) + \varepsilon_t \tag{2}$$

The state-space representation of this equation is:

$$F : \begin{bmatrix} x_{t-L} \\ x_{t-2L} \\ \vdots \\ x_{t-mL} \end{bmatrix} \to \begin{bmatrix} x_t = f(x_{t-L}, x_{t-2L}, \ldots, x_{t-mL}) + \varepsilon_t \\ x_{t-L} \\ \vdots \\ x_{t-mL+L} \end{bmatrix} \tag{3}$$

where ε_t represents the added noise, and its magnitude is measured by its variance. A noise-free system has $Var(\varepsilon_t) = 0$. L is the *time delay* and its introduction allows the possibility of skipping samples during the reconstruction. The parameter m is the *embedding dimension* or the length of past dependence. Since the dynamics are unknown, we cannot reconstruct the original map F. Instead, we search for an embedding space where we can reconstruct the map from the observed data that preserves the invariant characteristics of the original unknown map. The Jacobian-based approach can give consistent estimates of the Lyapunov exponent even in the presence of noise [22]. It consists of computing the Jacobian matrix of the chaotic map F, Eq. (4).