



Existence of spatial patterns in a predator–prey model with self- and cross-diffusion



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ABSTRACT

In this work, we investigate the spatiotemporal dynamics of reaction–diffusion equations subject to cross-diffusion in the frame of a two-dimensional ratio-dependent predator–prey model. The conditions for diffusion-driven instability are obtained and the Turing space in the parameters space is achieved. Moreover, the criteria for local and global asymptotic stability of the unique positive homogeneous steady state without diffusion are discussed. Numerical simulations are carried out in order to validate the feasibility of the obtained analytical findings. Different types of spatial patterns through diffusion-driven instability of the proposed model are portrayed and analysed. Lastly, the paper finishes with an external discussion of biological relevance of the analysis regarding cross-diffusion and pattern issues.

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1. Introduction

To study the population dynamics of interacting populations, a functional response of the predator to the prey density refers to the change in the density of prey per unit time per predator as a function of the prey density. Michaelis–Menten or Holling type II, an important and fashionable functional response is of the form

$$f(u) = \frac{pu}{mu + n},$$

where $p > 0$ is the maximal growth rate of the predator and $m, n > 0$ are the half-saturation constants. As $f(u)$ depends exclusively on prey density, it is commonly known as prey-dependent Michaelis–Menten or Holling type II response function [1–4]. Now-a-days, both ecologists and mathematicians are concentrated their attention to the ratio-dependent Michaelis–Menten-type or Holling type II functional response of the form

$$f\left(\frac{u}{v}\right) = \frac{p\left(\frac{u}{v}\right)}{m\left(\frac{u}{v}\right) + n} = \frac{pu}{mu + nv},$$

where numerical and functional responses depend on the sizes of both predator and prey species, especially when predators have to search for food and consequently have to share or compete for food. Modern field ecologists are interested in the ratio-dependent predator–prey models where predator–prey interactions have to be taken into account by the process of predation search [5–10]. Basically Holling type II functional response is mathematically and mechanistically very simple. The Holling type II function can be obtained mechanistically and requires the natural assumption that a predator cannot eat an unlimited amount of prey per unit of time. This simplicity allows the Holling type II functional response to be employed in a wide variety of models, ranging from simple two-dimensional models with one predator and one prey species

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to higher dimensional models involving, say, n_1 prey and m_1 predator species [11]. Modern ecological evidences suggest that ratio-dependent functional response is more appropriate when predators have a highly competitive searching process [12,13].

Studies on mechanisms and scenarios of spatiotemporal pattern formation in a system of interacting populations play an important role in modern theoretical ecology. Also the studies of spatiotemporal dynamics of predator–prey models have continued to be a vital issue to modellers since the pioneering work of Alan Turing [14]. A remarkable model for spatial pattern formation was introduced in his study whereby a statement was put forward by him that a set of chemicals reacting and diffusing throughout the tissues may exhibit spatial patterns under proper circumstances. In the theory of spatial pattern formation via Turing instability, it has been made known that a nonlinear system is asymptotically stable in the absence of diffusion but unstable in the presence of diffusion. This remarkable idea was destined to introduce a new scientific paradigm. Turing instability in reaction–diffusion system has been played as a mechanism for pattern formation in numerous embryological and ecological contexts [9,15–22]. In recent times, reaction–diffusion mechanisms form perhaps the most widely studied class of models for biological pattern formation and have been successfully applied to many patterning phenomena in modern ecology, biology, and many other fields of science.

In the history of population biology, the importance of the stability behaviour of a system of ratio-dependent Holling type II predator–prey model by taking into account the effect of self as well as cross-diffusion has received much less attention in the literature than other models, though interest is growing in the recent time for its spontaneous application in various branches of modern science. In recent years, many authors pay their attention to investigate the stability behaviour of a system of interacting populations by taking into account the effect of self as well as cross-diffusion [23–29]. Keeping all these in mind, an attempt is made here to investigate the role of cross-diffusion co-efficients on the spatiotemporal dynamics of a predator–prey model with ratio-dependent Holling type II functional response. Efforts have also been made to study the Turing pattern formation in the predation model where predator has an alternative source of food which may have an important role in promoting the persistence of predator–prey systems [5].

The paper is organised as follows: In Section 2, we introduce the ratio-dependent predator–prey model with diffusion and its non-dimensionalisation followed by the biological meanings of the parameters involved. In Section 3, we briefly revisit various properties for non-spatial case. The stability of the diffusive model alongwith the mathematical expression for Turing space has been discussed in Section 4. Section 5, illustrates the emergence of Turing patterns via numerical simulations. Finally in Section 6, the paper ends with some conclusions and comments.

2. Mathematical model and its analysis

The non-spatial ratio-dependent predator–prey model with Holling type II functional response is given by the following set of first-order non-linear ordinary differential equations:

$$\frac{du}{dt} = au \left(1 - \frac{u}{k_1} \right) - \frac{puv}{mu + nv} = g_1(u, v), \tag{2.1a}$$

$$\frac{dv}{dt} = bv \left(1 - \frac{v}{k_2} \right) + \frac{epuv}{mu + nv} - dv = g_2(u, v), \tag{2.1b}$$

$$u(0) > 0, \quad v(0) > 0, \tag{2.1c}$$

where u and v stand for the prey and predator density respectively at time t ; a , k_1 , b , k_2 , p , e ($0 < e < 1$) and d are all positive constants that stand for prey intrinsic growth rate, carrying capacity of the prey species, predator intrinsic growth rate, carrying capacity of the predator species, predation rate, conversion factor and predator death rate respectively; m and n are the half-saturation constants. To define the model (2.1) at the origin, we adopt the technique of Xiao and Ruan [19]. We now proceed to consider the spatial ratio-dependent predator–prey system in which the spatial aspects is described by diffusion. Incorporating the diffusion terms into Eq. (2.1), we arrive at the following spatially explicit system:

$$\frac{\partial u(t, x, y)}{\partial t} = au \left(1 - \frac{u}{k_1} \right) - \frac{puv}{mu + nv} + D_{11} \nabla^2 u + D_{12} \nabla^2 v, \tag{2.2a}$$

$$\frac{\partial v(t, x, y)}{\partial t} = bv \left(1 - \frac{v}{k_2} \right) + \frac{epuv}{mu + nv} - dv + D_{21} \nabla^2 u + D_{22} \nabla^2 v, \tag{2.2b}$$

$$u(0, x, y) > 0, \quad v(0, x, y) > 0, \quad \text{where } \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{2.2c}$$

represents the usual Laplacian operator. The parameters D_{22} , D_{11} are the positive self-diffusion coefficients while D_{12} , D_{21} are the cross-diffusion coefficients of the predator and the prey species respectively. Usually, diffusion is considered as a spatial transmission way, which moves from high concentration to low concentration and biologically, cross-diffusion means that the prey species exercise a self-defense mechanism to protect themselves from the attack of the predator. The value

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