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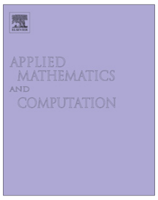

Effect of harvesting, delay and diffusion in a generalist predator–prey model ☆

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ABSTRACT

In compared with specialist predators which feed almost exclusively on a specific species of prey, generalist predators feed on many types of species. Consequently, their dynamics is not coupled to the dynamics of a specific prey population, and the generalist predators has itself growth function which be extended a well-known logistic growth term. We develop a generalist predator–prey model with diffusion and study the effect of harvesting and delay under Neumann conditions. The stability of the equilibria is firstly investigated, and the existence of traveling wave solutions is then established by constructing a pair of upper–lower solutions and using the cross iteration method and Schauder's fixed point theorem.

1. Introduction

In a natural ecosystem, predation is an important type of interaction between species and their living environment. In general, there are two main types of predators – generalist and specialist. Specialist predators feed almost exclusively on one species of prey [1–3]. Consequently, the predator's numbers are strongly dependent on the prey numbers, and the extinction of the prey will almost surely lead to the extinction of the predator. Lv et al. [4] studied the type of predator–prey relationship that is modeled by the following Lotka–Volterra model with harvesting

$$\begin{cases} \frac{du}{dt} = ru\left(1-\frac{u}{k}\right) - H(u,\ v) - c_1Eu, \\ \frac{dv}{dt} = cH(u,\ v) - dv - c_2Ev, \end{cases} \tag{1.1}$$

where $u,\ v$ denote the density of the prey and the predator, respectively. Here, parameters r is the intrinsic growth rate and k is the environmental carrying capacity of the prey; $H(u,\ v) = \frac{\beta uv}{\alpha+u}$ is Holling type II functional response; c is the ratio of biomass conversion (satisfying the obvious restriction $0 < c < 1$) and d is the natural death rate of the predator; c_1 and c_2 are the catchability coefficients of the two species, and E is the harvesting effort.

But, the generalist predator is quite different. Generalist predators feed on many types of species. When a focal prey population is threatened by extinction, the predator is capable of changing his diet to another species and may continue to persist. Therefore, their dynamics is not influenced by the dynamics of a specific prey population [1,5–8]. For example, two species of aphids are predominant in small grain crops: the English grain aphid and the oat-bird cherry aphid. Aphids high reproductive rate enables their populations to quickly build up to levels that can cause an economic loss. However, aphids

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are usually kept in check by biological control agents, such as lady beetles, parasitic wasps, and syrphid fly maggots, which are often abundant in small grain crops. Therefore, it is necessary and biologically desirable to investigate the type of generalist predator.

From this, we know that the generalist predator has itself growth rate function. In this paper, the growth function which represents the total number of the predator population v who born at τ time ago and survive now, is also extends the well-known logistic quadratically regulated death processes, i.e.,

$$be^{-d_1\tau}v(t-\tau)-\eta v^2.$$

The term which is first proposed by Aiello and Freedman [9] will be further explained as follows. In fact, parameter τ is the time delay which is the time from birth to maturity. As we know, delay differential equations exhibit much more complicated dynamics since a time delay could cause a stable equilibrium to become unstable and cause the populations to fluctuate [10–12]. Time delay due to gestation is a common example, because generally the consumption of prey by the predator throughout its past history governs the present birth rate of the predator. Therefore, more realistic models of population interactions should consider the effects of time delay. Gan et al. [13] were concerned with the existence of traveling wave solutions to a three-species Lotka–Volterra food-chain model with spatial diffusion and time delay.

On the other hand, species have the natural tendency to move from the areas of bigger population concentration to those of smaller population concentration. Note that spatial diffusion makes a significant contribution to the plentiful dynamics of ecological populations, especially to the propagation of species. Recently, it is an interesting topic that how does the balance of ecosystems persist via the long time interaction between prey and predator in habitation. There are some pioneering works to say about the following modified system (1.1) as follows

$$\begin{cases} \frac{\partial u}{\partial t}=D_1u_{xx}+ru(1-\frac{u}{k})-H(u,\,v), \\ \frac{\partial v}{\partial t}=D_2v_{xx}+cH(u,\,v)-d_2v. \end{cases} \tag{1.2}$$

The existence of traveling waves including the point-to-point orbits, periodic orbits (Hopf bifurcation) and point-to-periodic heteroclinic orbits of the model with Holling type I in [14,15], Holling type II in [16–19], Holling type III in [20] and general Holling type response in [21,22], is discussed by using the Wazewski Theorem together with a Lyapunov function and LaSalle's Invariance principle. Traveling waves can well model the oscillatory phenomenon and the propagation with finite speed of nature. In biology, traveling wave solutions described that the species invaded. As we know, predation is a common interaction between different species inhabiting the same environment. How species move, distribute, and persist is an important biological and mathematical question. The existence of traveling wave solutions for spatial systems provides a good answer to this question. It was shown that the predator and prey species can spread to expand their spatial ranges at the same speed. For example, the traveling waves connect the trivial and non-trivial steady states discussed in this paper demonstrate simultaneous invasions of both species into an unoccupied environment. The small invasion of the both species drives the system to a new stable of co-existence of them. The main objective of this paper is to study the existence of traveling wave solutions of a generalist predator–prey model with delay and harvesting by using the cross iteration method and Schauder's fixed point theorem.

Taking into account the generalist predator and time delay for reaction–diffusion systems, some authors [22–25] obtained the traveling wave propagation, as well as its precisely asymptotic behavior. Note that in these models in [23–25] and the references therein, the generalist predator species has a logistic growth function $v(r-bv)$ or $v(r-bv/(u+k))$, this implies that the predator has other food sources. In [9,26], authors considered the growth rate function $be^{-d_1\tau}v(y,\,t-\tau)$ for a predator–prey diffusion system. As mentioned in the literature [27], one type of the growth rate function of the generalist predator is given by

$$be^{-d_1\tau}\int_{-\infty}^{+\infty}G(x,\,y,\,\tau)v(y,\,t-\tau)dy-\eta v^2,\quad \text{where}\quad G(x,\,y,\,\tau)=\frac{1}{\sqrt{4\pi D\tau}}e^{-\frac{(x-y)^2}{4D\tau}}, \tag{1.3}$$

where b, d_1 and D are the birth rate, death rate and diffusivity of the immature predator species. If $D>0$, expression (1.3) allows for the fact that an individual that enters the mature predator class at position x will most likely have been born at some other point y. So, according to the assumptions about births and deaths made, the total rate of entering the mature predator class at time t and position x is therefore now made up as follows,

$$\begin{aligned} &bv(y,\,t-\tau)=\ \text{number bornat } y,\\ &e^{-d_1\tau}bv(y,\,t-\tau)=\ \text{number born at } y \text{ still alive now},\\ &\tfrac{1}{\sqrt{4\pi D\tau}}e^{-\frac{(x-y)^2}{4D\tau}}e^{-d_1\tau}bv(y,\,t-\tau)=\text{number born at } y \text{ still alive and now at } x \end{aligned}$$

and finally the integral totals up the contributions from all parts of the domain. Note that of those individuals born at y and still alive, to calculate the number that are now at x, we are multiplying by $G(x,\,y,\,t=\tau)$, where $G(x,\,y,\,t)$ satisfies the linear diffusion equation $\partial G/\partial t=D\partial^2G/\partial x^2,\ -\infty<x<\infty$, with the initial condition $G(x,\,y,\,0)=\delta(x,\,y)$. These arguments reflect the assumptions we have made about the diffusivity being linear diffusion.

Since the following population model we considered is in a closed environment, for convenience, we consider a finite domain $x\in(0,\,\pi)$. In ecosystem, the immature predators are concealed in the mountain cave and raised by their parents; they