Applied Mathematics and Computation 226 (2014) 406–414

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Shape ellipticity from Hu moment invariants

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ARTICLE INFO

Keywords:
Shape
Moment invariants
Shape descriptors
Shape ellipticity
Image processing

ABSTRACT

In this paper we derive an explicit formula which uses the first two Hu moment invariants to compute a shape ellipticity measure, i.e. to evaluate how much a planar shape differs from an ellipse. The ellipticity measure computed by this formula is invariant with respect to translation, rotation and scaling transformations. Also, the highest possible value is obtained if and only if the shape considered is an ellipse. Several experiments are also provided to confirm the theoretical observations.

A by product, of the derivations made in this paper, is an implicit interpretation of geometric/shape meaning of the second Hu moment invariant. A formula which connects the shape ellipticity and the first Hu moment invariant (both having a well understood behavior) and the second Hu moment invariant is derived.

1. Introduction

Image technologies develop rapidly. Huge amount of image based data becomes available in many domains, from medicine [1] to astrophysics [2] and from geology [3] to ecology [4]. Different object appear on images and they should be classified, recognized or identified. Different computing techniques were employed to solve such problems. In this paper we consider a shape based approach. The shape is one of the basic object properties, like texture and colour for example. Since the shape has many attributes/descriptors (e.g. compactness, elongation, convexity, etc.) which can be characterized numerically in many ways, it is very suitable to be explored in the tasks mentioned above. The basic idea is to assign several numerical characteristics, which are computed from shape descriptors, to the objects considered. Then one can compare objects based on these numerical characteristics. These numerical characteristics are used as coordinates of, so called, feature vectors which are then assigned to objects. Comparison in vector space is easy and straightforward in computer supported tasks. Generally speaking, a higher dimension of the feature vector used (i.e. more numerical characteristics assigned to the original object) provides a higher discrimination capacity between objects. This is the reason for an increasing demand for numerical evaluation methods of shape attributes/descriptors. Many techniques were used to define such methods, i.e. shape measures. So far: algebraic invariants [5], Fourier analysis [3], integral transformations [6], geometric reasoning [7], fractal techniques [8], logic [9], combinatorial methods [10], multiscale approaches [11], integral invariants [12], etc.

Shape measures which correspond to intuitively clear shape descriptors are of a particular interest. This is because the behavior of such measures (i.e. numerical shape characteristics) can be relatively easily understood and their behavior can be predicted to some extent. This is always an advantage because the suitability of such measures, for certain application, can be predicted a priori. For common shape descriptors, which are in the most frequent use, multiple measures, for their evaluation, are already designed. Just to mention a few: convexity [6,13,14], compactness [15–17], linearity [18–20],

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http://dx.doi.org/10.1016/j.amc.2013.10.062

ellipticity [16,21,22], etc. It is worth mentioning that multiple measures to evaluate a single shape property, are needed because there is no a shape measure which performs well in all applications. A measure which performs well in one application can fail to achieve the expectations in another.

There are also some generic shape measures (Fourier descriptors [3,23], moment invariants [5,24] or shape-illumination invariants [25]) which are not originally designed to measure a certain shape property/characteristic. This papers employs the first two Hu moment invariants [5], in order to design a shape ellipticity measure – a quantity which should indicate how much a shape differs from an ellipse. Although they have been introduced more than 50 years ago, as rotational shape invariants, and ever since used intensively in many image processing and computer vision tasks, the behavior of the Hu moment invariants has not been fully understood. It has been shown recently [7] that the Hu moment invariants are actually geometric invariants. The next step forward has been done in [17], where it has been proven that the first Hu moment invariant is minimized by circular shapes and maximized by shapes of very linear structure. Shape based explanation of the behavior the remaining of the Hu moment invariants is not know yet. In that sense, the formula derived here could be understand as the first step forward in the understanding of the second Hu moment invariant's behavior. This formula establishes a relationship between the second Hu moment invariant, first Hu moment invariant (whose behavior is well understood [17]), and an ellipticity shape measure, whose behavior also has a clear "shape" dependence.The paper is organized as follows. In the next section we give the necessary definitions and denotations. In Section 3 we derive the main result of the paper – a new ellipticity measure, computable from the first two Hu moment invariants. Several experiments are in Section 4. Concluding remarks are in Section 5.

2. Preliminaries

We start with definitions necessary to derive the main result of the paper – a new shape ellipticity measure, computable from the first two Hu moment invariants. First, we define the, so called, *geometric moments*, $m_{p,q}(S)$, of a planar region/shape S (see [26]):

$$m_{p,q}(S) = \int\int_S x^p y^q dxdy. \tag{1}$$

Throughout the paper, even not mentioned, all appearing shapes S will be normalized in a standard way:

- All shapes are translated such that their centroid and the origin coincide;
- All shapes are scaled such that their area becomes equal to 1, i.e. such that $m_{0,0}(S) = 1$.

Such an shape normalization is not a restriction in applications since the shape of an object does not change under translation and scaling transformations.

An additional assumption, irrelevant to the applications since we are dealing with area based descriptors, will also be made. We will say that two shapes S_1 and S_2 are equal if and only if their set differences have the area equal to zero, i.e. if $Araea\_of\_(S_1 \setminus S_2) = Araea\_of\_(S_2 \setminus S_1) = 0$, e.g. the open circle $S_1 = \{(x,y) \mid x^2 + y^2 < 1\}$ and the closed circle $S_1 = \{(x,y) \mid x^2 + y^2 \leqslant 1\}$ are of the same shape.The moments $m_{0,0}(S), m_{1,0}(S)$, and $m_{0,1}(S)$ are used to define the centroid $\left(\frac{m_{1,0}(S)}{m_{0,0}(S)}, \frac{m_{0,1}(S)}{m_{0,0}(S)}\right)$ of a given shape S [26]. Since we assume that all shapes appearing are of unit area, the centroid of S can be expressed as $(m_{1,0}(S), m_{0,1}(S))$. The new ellipticity measure, will be derived by using the first two Hu moment invariants, $\mathcal{I}_1(S)$ and $\mathcal{I}_2(S)$, which are defined, [5], as:

$$\mathcal{I}_1(S) = m_{2,0}(S) + m_{0,2}(S), \tag{2}$$

$$\mathcal{I}_2(S) = (m_{2,0}(S) - m_{0,2}(S))^2 + 4 \cdot m_{1,1}(S)^2. \tag{3}$$

Note. Again, $m_{0,0}(S) = 1$ is assumed in the formulas above.

It is worth mentioning that moment invariants have already been used to measure shape ellipticity. Affine invariant $\mathcal{J}(S)$, defined in the reference [27] as

$$\mathcal{J}(S) = (m_{2,0}(S) \cdot m_{0,2}(S) - m_{1,1}(S)^2)/m_{0,0}(S)^4 \tag{4}$$

has been used in [21] to define the ellipticity measure $\mathcal{E}_I(S)$ in the following way:

$$\mathcal{E}_I(S) = \min\{16\pi^2\mathcal{J}(S),\ (16\pi^2\mathcal{J}(S))^{-1}\}. \tag{5}$$

The triangularity measure, from the same paper, is also based on $\mathcal{J}(S)$. Both ellipticity and triangularity measures, from [21], are adopted to range over $[0, 1]$ and peaking at 1 for a perfect ellipse (perfect triangle). The problem is that, for both measures, if the measured ellipticity (triangularity) equals 1, it is not guaranteed (or at least not proved) that the considered shape is a