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Generalized exponential chain ratio estimators under stratified two-phase random sampling

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ABSTRACT

In this study, we have considered stratified two-phase sampling scheme for estimating the population mean of study variable taking auxiliary information. The proposed chain estimators are the exponential function of auxiliary variables. The mean square errors and biases equations have been obtained for the proposed estimators. Further, the proposed estimators are given in generalized form. The conditions for which proposed estimators are more efficient as compared to other estimators have been discussed. It is shown that the proposed estimators are more efficient as compared to the unbiased sample mean estimator, modified stratified two-phase ratio and product estimators. Empirical study has also been carried out to demonstrate the performance of proposed estimators.

1. Introduction

In survey sampling, the auxiliary information is often considered to improve the efficiency of estimator for the finite population mean. It is usual to estimate unknown population mean of study variable using ratio or product estimator for positive or negative correlation between study variable and auxiliary variable respectively. Stratification of population is a probability sampling design that is used to increase the precision of estimation. In stratified sampling design, the population under study is divided into homogenous subgroups called strata and probability sampling is applied within each strata. Kadilar and Cingi [5] used the Upadhyaya and Singh [19] estimator in stratified random sampling. Kadilar and Cingi [6], Shabbir and Gupta [14,15] have suggested ratio estimators using stratified sampling scheme.

Chand [2] introduced the chain estimator by considering the information from two auxiliary variables. The estimators for the population mean in double sampling by considering an additional auxiliary variable have been discussed by Kiregyera [7], Kiregyera [8], Sahoo and Sahoo [11], Sahoo et al. [12], Samiuddin and Hanif [13], Singh et al. [18] and Singh and Choudhury [16]. The exponential estimators may be used for the exponential relationship between study variable and auxiliary variable. Bahl and Tuteja [1] suggested the exponential estimator under simple random sampling without replacement. Singh and Vishwakarma [17], Noor-ul-Amin and Hanif [10] have proposed exponential type estimators under double sampling scheme. The aim of this paper is to propose the exponential type estimators under stratified two-phase sampling scheme.

2. Stratified two-phase random sampling

Consider a finite population of size N which is stratified into L homogenous strata. Let N_h be the size of hth stratum ($h = 1, 2, ..., L$) such that $\sum_{h=1}^{L} N_h = N$ and (y_{hi}, x_{hi}) be the observations of the study variable (y) and the auxiliary variable

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(x), on the ith unit of hth stratum, respectively. Moreover, $\bar{y}_h$ and $\bar{x}_h$ be the sample means of hth stratum corresponding to the population means $\bar{Y}_h$ and $\bar{X}_h$ of y and x respectively. When information on $\bar{X}_h$ is unknown, a first phase large sample of size n'_h is selected from each hth stratum to estimate $\bar{X}_h$ and then replace it with its unbiased estimator. To obtain the bias and mean square error (*MSE*) under stratified two-phase sampling, let us define

$$\left.\begin{array}{l} \bar{Y} = \sum_{h=1}^{L} P_h\bar{Y}_h, \quad \bar{y}_h = \bar{Y}_h(1+e_{oh}), \quad e_0 = \frac{\sum_{h=1}^{L} P_h\bar{Y}_h e_{0h}}{\bar{Y}}, \\ \bar{X} = \sum_{h=1}^{L} P_h\bar{X}_h, \quad \bar{x}' = \bar{X}_h(1+e'_{1h}), \quad e'_1 = \frac{\sum_{h=1}^{L} P_h\bar{X}_h e'_{1h}}{\bar{X}}, \quad \bar{x}_h = \bar{X}_h(1+e_{1h}), \quad e_1 = \frac{\sum_{h=1}^{L} P_h\bar{X}_h e_{1h}}{\bar{X}}, \\ \bar{Z} = \sum_{h=1}^{L} P_h\bar{Z}_h, \quad \bar{z}' = \bar{Z}_h(1+e'_{2h}), \quad e'_2 = \frac{\sum_{h=1}^{L} P_h\bar{Z}_h e'_{2h}}{\bar{Z}}, \quad \bar{z}_h = \bar{X}_h(1+e_{2h}), \quad e_2 = \frac{\sum_{h=1}^{L} P_h\bar{Z}_h e_{2h}}{\bar{Z}} \end{array}\right\}. \tag{2.1}$$

Using the notations given in (2.1), expectations are defined as,

$$\left.\begin{array}{ll} E(e_0) = E(e'_1) = E(e_2) = 0, & V_{r,\,s,\,t} = \sum_{h=1}^{L} P_h^{r+s+t} \frac{E((\bar{x}'_h-\bar{X}_h)^r(\bar{y}_h-\bar{Y}_h)^s(\bar{z}_h-\bar{Z}_h)^t)}{\bar{X}^r\bar{Y}^s\bar{Z}^t} \\ E(e_0)^2 = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} P_h^2\lambda_h S_{yh}^2 = V_{020} & E(e'_1)^2 = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} P_h^2\lambda'_h S_{xh}^2 = V'_{200} \\ E(e_1)^2 = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} P_h^2\lambda_h S_{xh}^2 = V_{200} & E(e_2)^2 = \frac{1}{\bar{Z}^2}\sum_{h=1}^{L} P_h^2\lambda_h S_{zh}^2 = V_{002} \\ E(e_0.e_2) = \frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{L} P_h^2\lambda_h S_{yzh} = V_{011} & E(e_0.e_1) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} P_h^2\lambda_h S_{xyh} = V_{110} \\ E(e_0.e'_1) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} P_h^2\lambda'_h S_{xyh} = V'_{110} & E(e'_1.e_2) = \frac{1}{\bar{Z}\bar{X}}\sum_{h=1}^{L} P_h^2\lambda'_h S_{xzh} = V'_{101} \\ \lambda_h = \left(\frac{1}{n_h} - \frac{1}{n'_h}\right) \quad \lambda'_h = \left(\frac{1}{n'_h} - \frac{1}{N_h}\right) & \Lambda_h = \lambda_h - \lambda'_h \\ `\vartheta_{002} = V_{002} - V'_{002} \quad \vartheta_{011} = V_{011} - V'_{011} & \vartheta_{200} = V_{200} - V'_{200} \quad \vartheta_{110} = V_{110} - V'_{110} \end{array}\right\}. \tag{2.2}$$

The procedure of stratified two-phase sampling is as follows:

i. Select a sample of size n'_h from the hth stratum using simple random sampling without replacement (*SRSWOR*) such that $\sum_{h=1}^{L} n'_h = n'$ and observe auxiliary characteristic(s) for these units. This is called a stratified first-phase sample.
ii. Select another stratified random sample of size n_h from each n'_h ($n_h < n'_h$) using SRSWOR such that $\sum_{h=1}^{L} n_h = n$ and collect information on variable of the interest say y. This is called a second-phase sample.

Under stratified two-phase sampling, usual unbiased estimator for population mean $\bar{Y}$ is given by

$$\bar{y}_{st} = \sum_{h=1}^{L} P_h\bar{y}_h \quad \text{where} \quad P_h = \frac{N_h}{N}. \tag{2.3}$$

The estimator (2.3) is unbiased with variance,

$$Var(\bar{y}_{st}) = \bar{Y}^2\left(\frac{\sum_{h=1}^{L} P_h^2\lambda_h S_{yh}^2}{\bar{Y}^2}\right) = \bar{Y}^2 V_{020} \tag{2.4}$$

We consider ratio and product estimators under stratified two-phase sampling as,

$$\hat{\bar{Y}}_{Rd} = \bar{y}_{st}\frac{\bar{x}'_{st}}{\bar{x}_{st}} = \sum_{h=1}^{L} P_h\bar{y}_h\left(\frac{\sum_{h=1}^{L} P_h\bar{x}'_h}{\sum_{h=1}^{L} P_h\bar{x}_h}\right), \tag{2.5}$$

and

$$\hat{\bar{Y}}_{Pd} = \bar{y}_{st}\frac{\bar{x}_{st}}{\bar{x}'_{st}} = \sum_{h=1}^{L} P_h\bar{y}_h\left(\frac{\sum_{h=1}^{L} P_h\bar{x}_h}{\sum_{h=1}^{L} P_h\bar{x}'_h}\right). \tag{2.6}$$

The mean square errors of above estimators, up to first order approximation are,

$$MSE(\hat{\bar{Y}}_{Rd}) = \bar{Y}^2(V_{020} + \vartheta_{200} - 2\vartheta_{110}), \tag{2.7}$$

$$MSE(\hat{\bar{Y}}_{Pd}) = \bar{Y}^2(V_{020} + \vartheta_{200} + 2\vartheta_{110}). \tag{2.8}$$