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Stability analysis and optimal control of pine wilt disease with horizontal transmission in vector population

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ABSTRACT

In this paper, we have proposed and mathematically modeled an epidemic problem with vector-borne disease. We have taken three different classes for the trees, namely susceptible, exposed and infected, and two different classes for the vector population, namely susceptible and infected. In the first part of our paper, we rigorously analyze our model using the dynamical systems approach. Global stability of equilibria is resolved by using Lyapunov functional. In the second part, the model is reformulated as an optimal control problem in order to determine the significance of certain control measures on the model. We apply four control parameters, namely the tree injection control to the trees, deforestation of infected trees, eradication effort of aerial insecticide spraying and the effort of restrain of mating. Both numerical and analytical methods are employed to ascertain the existence of cost effective control measures.

1. Introduction

Infectious diseases are caused by pathogens such as bacteria, protozoa, and viruses. These pathogens may reach the healthy organism by disease causing biological agents, known as vectors (e.g insects and ticks), who carry the pathogen from infected individuals and transmit it to the uninfected ones. Infectious diseases which spread by means of vectors are known as vector-borne diseases, and these effect all living organisms, including plants. Examples of vector-borne infections in trees include the pine wilt disease and the red ring disease in palms which are caused by nematodes (roundworms), and are vectored by insects [1]. Bursaphelenchus xylophilus is the nematode which causes the pine wilt disease (PWD) [2,3]. The pine sawyer beetle (or Monochamus alternatus) is the vector for PWD. This beetle scatters the nematode, from space, over healthy pine trees. However, direct transmission in vector population also occurs during mating [4,5]. Reproductively mature beetles use twig bark of healthy trees for feeding purpose, and concentrate on diseased trees for copulation and oviposition [6]. Horizontal transmissions of Bursaphelenchus xylophilus between heterosexual vectors enhances the level of multiple infections. The first epidemic of PWD occurred in Japan in 1905. Except for the northern districts of the country, this disease had spread in all of Japan by the 1970s [7]. After another ten years, the PWD epidemic had spread to many parts of Asia, such as China, Taiwan, Hong Kong and Korea. In 1999, the disease hit Europe (Portugal) [8]. Today, the PWD is one of the leading threats to forest ecosystems throughout the world.

Mathematical modeling is useful in understanding the process of transmission of a disease, and determining the different factors that influence the spread of disease. In this way, different control strategies can be developed to limit the spread of infection. Lately, some mathematical models have been formulated on pest-tree dynamics, that have given

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successful results [9], such as a PWD transmission model was investigated by Lee and Kim [10]. This model incorporated non linear incidence rates. In our paper, we have proposed a mathematical model which describes the host-vector relationship between pine trees and pine sawyer beetles (carrying nematode) by means of ordinary differential equations with bilinear incidence rate. That is, a vector-host epidemic model with horizontal transmission in vector population is presented, where the dynamics of the host pine trees and vector beetles are described by *SEI* and *SI* models, respectively. The research work done previously in this area contains some good papers on the vector diseases with direct transmission in host population [11–13]. Some mathematical work about the transmission dynamics of vector-borne disease have been published recently [14–16]. Few researchers have also applied optimal control methods to limit the virus transmission [17]. Moulay et al. [18] investigated the impact of three time dependent control variables, namely destruction of breeding sites, prevention, and treatment efforts. Okusun et al. [19] used vaccination and treatment control of a malaria disease transmission model.

The purpose of this paper is two fold. The first is to carry out the stability analysis of the proposed model. Detailed study of the model reveals that there exist two equilibria; the disease-free equilibria and the endemic equilibria. In order to prove the global asymptotical stability, we have used the Lyapunov function theory. It is proved that the global dynamics are completely determined by the basic reproduction number. The second aim is to achieve awareness about the most desirable technique for minimizing the transmission of PWD with horizontal transmission in vector population using the optimal control theory. To do this, we consider an optimal control model with four time-dependent controls: tree-injection u_1, deforestation of infected trees u_2, aerial pesticide spraying u_3 and the effort of restrain of mating u_4. We carry out further analysis and deduce those conditions where it is optimal to totally eliminate, and not just control, the PWD. In those cases where eradication is unattainable, we have given the necessary conditions for optimal control of the PWD using Pontryagin's Maximum Principle.

The paper is organized as follows. The model is formulated in Section 2. Existence and global stability of the equilibria of the model are investigated in Section 3. In Section 4, the model is extended and an optimal control problem is formulated. In Section 5, the existence of an optimal control is examined, then, the optimality system is derived, which characterizes the optimal control. Section 6 provides numerical simulations for the optimal control model. Lastly, we give a brief discussion of our results in Section 7.

2. The ODE model

In this section, we formulate a mathematical model for pine wilt disease in the pine and beetles population with total population size at time t given by $N_h(t)$ and $N_v(t)$, respectively. The total host population is partitioned into three subclasses, namely susceptible pine trees, $S_h(t)$, exposed pine trees, $E_h(t)$, and infected pine trees, $I_h(t)$. Susceptible host pines are those trees which are healthy and have the potential to be infected by the nematode. Healthy trees emit oleoresin, which acts as a physical barrier to beetle oviposition. Exposed host pine trees are the ones which have been infected by the nematode, but still possess the capability for oleoresin production. The infected host pine trees are those trees which have been infected by the nematode and have lost the ability to exude oleoresin, and hence, beetles can oviposit on them. Furthermore, it is assumed that the recovered class $R_h(t)$ is negligible because every infectious pine tree dies within the year of infection or in the next year.

The total vector population is subdivided into two subclasses: the susceptible adult beetles, $S_v(t)$, who do not carry pinewood nematode at time t, and the infective adult beetles, $I_v(t)$, who carry pinewood nematode at time t (i.e, when the adult beetles emerge from dead pine trees). Our model excludes the immature beetles which is eggs, a pupal stage because they do not participate in the infection cycle. The parameters used in the system are as follows: the parameter Λ_h is the constant increase rate of pine tree at time t and Λ_v is the constant emergence rate of adult beetles at time t during the period of emergence. The per capita natural death rate of pine trees and beetles (as vectors) is given by μ_h and μ_v, respectively. The parameter α_1 is the probability at which the infected beetles transmit nematodes by means of contact and α_2 denotes the rate at which the infected beetles transmit nematodes by oviposition. The parameter θ denotes the probability that susceptible host pine trees die a natural death, instead of being infected by the nematode. The nematode is transmitted to the pine trees through beetle feeding wounds and oviposition wounds. The rate of transmission of nematodes to the pine trees through beetle feeding wounds is denoted by α_1, so that the incidence of new infections through this route is given by the mass action term $\alpha_1 S_h I_v$. The rate of transmission through oviposition is denoted by α_2, and the incidence of new infections via this route is given by the mass action term $\alpha_2 \theta S_h I_v$. The parameter β denotes the transfer rate from the exposed to the infected class. Susceptible vectors can get infected through two routes: by getting infected with the pinewood nematode when the beetle emerge out in the infectious tree, and by coming in contact with an infected individual (for example, during mating). The probability of getting infected with the pinewood nematode when the beetle emerge out in the I_h class is denoted by γ_1, so that the incidence of new infections via this route is given by the mass action term $\gamma_1 S_v(t) I_h(t)$. Furthermore, the parameter γ_2 denotes the probability that the beetles get infected directly during mating, so that the incidence of new infections transmitted by the vectors is given again by a mass action term $\gamma_2 S_v(t) I_v(t)$. We have assumed that the pinewood nematode enters the healthy trees through feeding wounds made by the beetles. The following system of coupled nonlinear differential equations, derived on the basis of parameter definitions and assumptions, describe the dynamics of the PWD: